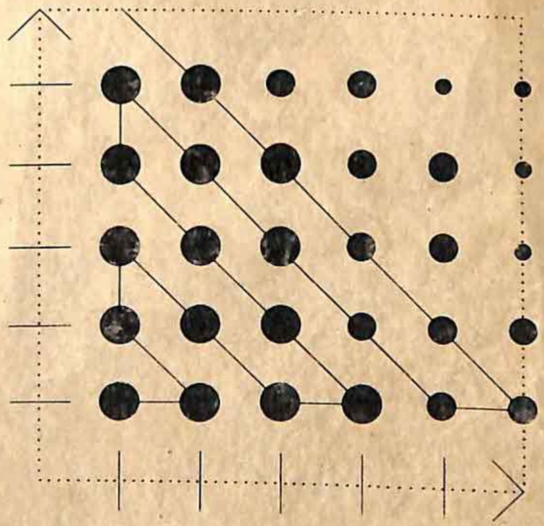
Volume 2

Studies in mathematics education

Edited by Robert Morris

The teaching of basic sciences Mathematics



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Studies in Mathematics Education

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Edited by Robert Morris

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Preface

INSTITUTE of EDUCATION
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Studies in Mathematics Education, Volume 2 is published in the Unesco series 'The Teaching of Basic Sciences', a series which also includes *New Trends in Mathematics Teaching*, (volumes I to IV, published in 1966, 1970, 1972 and 1978). The first volume of *Studies in Mathematics Education* was published in 1980. It is a collection of seven articles which describe developments in mathematics as a part of general secondary education in the schools of Hungary, Indonesia, Japan, the Philippines, the Union of Soviet Socialist Republics, the United Kingdom and the United Republic of Tanzania.

Significant reforms have been introduced in the teaching of mathematics since the 1960s, largely reflecting some of the changes in mathematics itself. Since these reforms, school populations have altered. A much wider spectrum of the population is now receiving instruction in mathematics. Mathematics programmes developed earlier are now being taught to this more diverse group of students. And, in many instances, these programmes do not seem to take into account the background and needs of the student population.

Another change is that mathematics has permeated society. Not only are there new professional users of mathematics, but the ordinary citizen is expected to understand more aspects of mathematics in his daily life. Thus the question: does the teaching of mathematics correspond to the needs of the majority of pupils and the society?

In search of an answer to this question, Unesco undertook a review of the goals of mathematics teaching, especially at school level, leading to guidelines for the design of future mathematics programmes. Studies were commissioned, and a meeting in May 1980 was convened in an attempt to answer the question. *Studies in Mathematics Education, Volume 2* is the result of these actions.

This volume includes chapters of three types: descriptions of goals reflecting some need of society; case studies of national goal-setting; and a summary of the meeting on goals for mathematics education.

In the first group, the papers examine goals as a reflection of the needs of society, in particular the needs for commerce, industry, agriculture and research, and, at the same time, the needs of the learner, assuming that the mathematics is taught as part of general education.

The national case studies examine strategies of goal assessment, and compare the intended, implemented and realized curriculum. It is here that the reader can find examples of national goals for mathematics teaching.

The discussions of the meeting have been summarized in the final chapter of the volume. The publication is intended to contribute to the design of future mathematics programmes. It is aimed primarily at mathematics educators and policy makers concerned with mathematics education.

This volume, as was the case with its predecessor, was edited by Robert Morris, formerly Director of the Schools Council Continuing Mathematics Project at the University of Sussex.

It is hoped that this volume will constitute a valuable contribution to the literature in the field. Unesco wishes to express its appreciation to Robert Morris and the various contributors to this second *Studies in Mathematics Education.* The views expressed are of course those of the authors. They do not necessarily represent any position on the part of Unesco or of the editor.

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Introduction

If the sixties were the years of fermentation in mathematics education, the seventies have been years of maturing thought in many countries about the suitability of the modernized curriculum. Some heart-searching has tempered the enthusiasm of the reformers. Some doubts have been voiced by lay people. Some users of mathematics have complained about waning skills. Concerns have touched on questions of content, questions of teaching method, questions of scope and questions of relevance. With this, it seemed to Unesco to be timely to take stock of evolving opinion, and, to review, in particular, the question which is at the heart of the curriculum – the question of goals.

This second volume of *Studies in Mathematical Education* is exclusively devoted to a discussion of goals. Its chapters have served two purposes: first, the purpose most evident to the reader – to comprise this book; second, a previous purpose – to provide background for those who took part in the Meeting of Experts on Goals for Mathematics Education: Mathematics Teaching and the Needs of Society in May, an account of which furnishes the material of the final chapter.

Goals have to take account of many needs: those of society in its urban, rural and suburban forms; those of the teacher; those of the learner; those, certainly, of the discipline itself. The various contributors have severally addressed themselves to these needs, and to the consequences for curricular emphasis. Other contributors have illuminated the processes whereby goals and objectives are formulated and then find their reflection in teaching materials. The collective result is a rich concoction of considerations and conclusions which evoke the prospect of an exciting decade of new endeavour in the unending effort to enrich the effectiveness of mathematical education.

The first chapter is perhaps the hardest. It is a penetrating insight into the 'purposes' and the 'objectives' of mathematical education. Each category is sub-divided, so that a four-fold relationship is seen to exist between the types of goals which characterize a typical curriculum.

The author then tests his analysis against twentieth century trends in the United States of America, the United Kingdom and in pre-war Germany.

The second chapter adopts the view-point of the learner, and makes the point that the mastery of a particular concept is a matter of progressing through a number of 'levels of understanding'. It describes research aimed at determinating these 'levels', the consequential discovery of a wide range of understanding in each age group in the secondary stage and of little difference in this range from year to year, a situation which only slowly improves with time, and which calls in question the conventional beliefs about teaching and learning.

The third chapter complements the first. Its thesis is that education for rural development implies an integration of school with the rural community. The implications for the mathematical component of the curriculum are then examined. Five categories of goals are found: social, technical, personal, cultural and aesthetic.

Chapters 4 and 5 make an interesting pair of contrasting contributions on the theme of mathematics for the world of work. Chapter 4, in particular, brings out the consequences for school leavers of the inadequacies of their teachers, notably the errors which many primary teachers perpetuate, and the inability of many secondary teachers to perceive the root cause of a learner's difficulty.

It is argued in Chapter 6, that the commonly assumed dichotomy between 'mathematics for mathematicians' and 'mathematics for non-mathematicians' is a false one, and that belief in its existence is dangerous and conducive to the continuity of bad teaching. Future professionals and future users both need good mathematics, the key to which is time: time to wrestle with problems, time to go up blind alleys, time to evolve *a* solution, not necessarily *the* solution.

Chapter 7, in reviewing the reforms of the last two decades, paves the way for Chapter 8, which describes an interesting attempt to find a consensus on the way forward into the eighties by polling a sample of interested parties on their preferences for emphasis in school mathematics.

The ninth chapter is revealing of what can be accomplished on a regional basis, given co-operation, goodwill and some Unesco assistance.

The penultimate chapter is salutary. It is a telling reminder of the fact that the intended goals of mathematics education can sometimes suffer unintended dilution and distortion at the hands of second and third parties to the teaching and learning process.

Mogens Niss

Goals as a Reflection of the Needs of Society

Introduction

Everything said in this contribution lies in the delicate and controversial twilight zone between mathematics, perceived as a subject, and education, regarded as a social phenomenon. It is difficult to manoeuvre in this zone. Mathematicians, mathematics educators and mathematics educationalists often feel a certain uneasiness about the political and idealogical facets with which social matters are inevitably associated. Or they feel inadequately qualified in social science. On the other hand, generalists in education only very seldom possess a sufficient knowledge of mathematics and mathematics teaching to consider satisfactorily all aspects of the complex of mathematics education. The author – a mathematician, a mathematics educator and a mathematics educationalist, risking his reputation by transgressing the preserves of mathematics and entering into considerations which derive from society – is not above the difficulty.

In fact, the problem of reconciling the conflicting claims is even worse than it seems at first sight to be. Having a rather general scope, this contribution ought to be of relevance to the situations in a multitude of countries, whether in the first, the second or the third world, with their different social, economic and political backgrounds, and including their highly centralized or their highly decentralized organizational systems as may be. Such a scope is in itself highly ambitious. It is difficult to do justice to. The author accordingly wishes to apologize for having been unable to emancipate his considerations altogether from the limited horizon induced by his specific national perspectives.

Issues of mathematics education can be considered under two different perspectives: an *analytic perspective* and a *normative perspective.* Considerations made under an analytic perspective concentrate on dissecting the existing state of affairs: identifying its fundamental components and determinants, the causal connections within it and the sources of influence upon it, etc. When considerations are made under a normative perspective, ideas, principles and suggestions, or whatever, are put forward and advocated, sometimes, perhaps, against a background of analytic considerations.

The present paper is primarily an analytical one. Normative considerations are based on values, including socio-political values. So, it would probably be futile to put forward such suggestions or imperatives for the goals of mathematics education in a publication of world-wide distribution. This does not mean that this paper claims to be neutral in the matter of values. Values lie at the very core of these matters. They influence analytic considerations also. It is, however, thought that a paper which is concerned with understanding general external factors of importance to the role and position of mathematics education will be more relevant in an international context than concrete proposals would be.

This article is mainly concerned with the mathematics education of those pupils and students (all henceforth termed 'students'), of 6 to 19 years of age, who progress through the official school system. It will not pay much attention to the very specialized courses of mathematics education designed for a narrow and selected clientele in specific institutions outside or on the fringes of the general school system.

Methodological Difficulties

When analysing the goals of mathematics education as a reflection of the needs of a given society, one is faced with the principal and practical problem of detecting and identifying (and documenting) the actual impact on mathematics education of social needs. Only seldom has 'society' articulated its general interests and motives directly and publicly. Most often, official and semi-official statements are confined to dealing, at a rather low level of generality, with the specific objectives of mathematical content, knowledge and skills. These, often, are oriented primarily towards examination requirements. And, in such statements, the ultimate ends of mathematics education are presupposed, or assumed to be irrelevant. Even if this were not the case, if 'society' were to articulate its interests, by espousing the ultimate ends of mathematics education, we could not be sure that the publicly articulated interests and 'the real, objective interests' of society (and what these are is subject to discussion) would coincide.

In this respect, the situation differs from society to society. In some societies, the fixing of general educational goals and curricula is strongly centralized and bears the stamp of political control. In other societies the planning is centralized, but the decisions are left to non-political professionals. In still other societies decisions on goals and on consequent curricula are delegated to the local level, or, in the extreme, to the individual school, or even to the individual teacher. Combinations of these features (for instance, a centrally designed framework, with certain decisions on content left to the teachers) are also frequently encountered.

The situation described implies that analyses of the reflection at issue are condemned to follow paths hidden partly in fog. Furthermore, it implies that conclusions deriving from these matters must necessarily be vague, unless they are to some extent advanced as postulates. To avoid obscuring the main lines of this paper too much by reservations and modifications, we have preferred postulates to vagueness.

In any society where mathematics education is being given, there are intimate relations between mathematics education on the one hand and the political, ideological and economic interests of that society on the other. It might very well be that these relations are not the result of a well-defined set of conscious considerations, backed up by administrative decisions and initiatives. But this does not imply that the relations do not exist. In the context of this paper, it probably would not be a reasonable investment of effort to attempt to prove the existence of such relations. A proof would have to consider, in a very concrete way, the different societies covered by the assertion. So, we shall confine ourselves to emphasizing that, in any society, mathematics education in school is part of the total school system of that society. Hence mathematics education is bound to be subject to the interests of society as reflected in the school system, whether those interests imply that school is allowed relative autonomy, with only funds provided, or whether they imply that school is subject, to some degree or another, to central control. Instead, it seems more relevant to our theme and purpose to investigate a little further the possible shapes and characteristics which the relations in question can take and have actually taken.

Since, in this connection, we are primarily interested in that subset of the total set of relations between mathematics education and society which influences the goals, objectives and purposes, it seems necessary to clarify the terms.

Terminological Distinctions: Goals, Purposes and Objectives

Throughout this paper we distinguish between *two kinds of goals*: 'the purposes' and 'the objectives'.

By the term 'purposes', we shall have in mind the *motives* and *reasons* for the mathematics education of a school system. These are the expected gains which justify the existence of an education in mathematics. In other words, to set out 'the purposes' is to answer the question 'why should mathematics education exist?'.

By the term 'objectives', we shall have in mind those *specific qualifications* (in terms of insight, knowledge, accomplishments, behaviour, attitudes, etc.) which the mathematics teaching is aiming to inculcate.

The above distinction will assist analysis. It is only very seldom that, in official documents, in commentaries and in debates on the goals of mathematics education, such a distinction is made between the 'ultimate ends' (or 'purposes') and the other kinds of 'ends'. So, in discussion, it becomes difficult to separate the strata which, in our opinion, are distinct. This can give rise to confusion, not only of terminology (since the terms 'goal', 'aim', 'purpose' and 'objective' are often used indiscriminately), but also in matters of substance. It should, of course, be confessed that the distinction here made is neither founded on theory nor is it sharp. But this deficiency does not affect the substance of the distinction which has been drawn. The fact that official documents do not distinguish between 'purposes' and 'objectives' does not imply that the distinction cannot be made, as the examples which follow will make clear.

Example 1: One might assert: 'It is a *purpose* of mathematics education to make it possible to utilize mathematical tools to contribute to technological and economic growth in society.' If so, a corresponding objective might be: 'It is an *objective* of mathematics education to train students to analyse human problems, to recast them in mathematical terms, to solve the resulting mathematical problems, and to re-interpret the mathematical solutions in such a way as will provide intelligible answers to the original problem.'

Example 2 goes along a different line. One might assert: 'It is a *purpose* of mathematics education to contribute to developing, in the individual citizen, general mental capacities of use to his own life, such as: the ability to reason logically and to assess the logical reasoning of others, imagination, inventiveness and aesthetic appreciation.' To this purpose, a corresponding objective could be this: 'It is an *objective* of mathematics education that students should acquire insight into the role, the nature and the construction of at least one axiomatic structure in mathematics.'

It is not, of course, possible to deduce the objectives of mathematics education from a declared purpose. The determination of the objectives depends on several factors. Of these, the most important are the level (including the age level) of the students thought of, the character and the needs of the society in question, the organizational framework within which mathematics education is provided, and the outlook upon mathematics and upon mathematics education which the curriculum planners collectively hold. For instance, corresponding to the 'purpose' of Example 1, quite different objectives could reasonably be adduced according to the level of the students involved or the needs of society. Here is one: 'It is an objective of mathematics education to make students understand and accept that some practical problems are susceptible to mathematical treatment.' Another might be: 'It is an objective of mathematics education to train students to perform arithmetical operations quickly and accurately.'

As one moves from a 'purpose' towards an 'objective' some generality of statement and social involvement is lost. One becomes more prescriptive. This process continues as we proceed to the more specific objectives, the content, the means and the methods of mathematics teaching. But the ultimate interests of society in mathematics education are entrenched in 'the purposes'. The objectives are the more detailed, precise and concrete specifications of the purposes to which they refer. In short, they are vehicles for the pursuit of purposes. What matters, when attempting to classify a goal as a purpose, is whether or not the goal is really a purpose as we have defined it to be. Whether, that is, it bears the stamp of being an ultimate end in its own right. The ambiguity which arises from the possibility of classifying certain goals either as purposes or as objectives (or both) is a source of obscurity in debates on mathematics education.

Types of Purposes

In most countries, mathematics has always been taught in some schools to some students. But very different (and sometimes, even, mutually incompatible) purposes are advanced to justify the existence of mathematics education. This phenomenon might seem to indicate that the existence of mathematics education is not, in fact, a function of the purposes. Or, it may be that 'purposes' are trivial, or that they have no more bearing upon mathematics education than the mottos of kings have had upon their actual reigns. But the role and character of mathematics education, its specific objectives, its content, the methods used and the students taught, are strongly influenced by the purposes. Moreover, it is of crucial importance, so far as the mathematics education of the masses of students is concerned, to identify, or, if need be, to establish the purposes.

Until recently, purposes have not been advanced to justify the teaching of arithmetic. The usefulness of arithmetic has always been considered self-evident in every society. Such difficulties as arise with purposes are to be found in the mathematics which comes later, though arithmetic teaching may not now be exempt from difficulties with the introduction and dissemination of pocket calculators. All in all, there seem to be good reasons for looking deeper into the purposes of mathematics education.

There are essentially two main types of purposes of mathematics education. One type concerns the needs of society as a whole. These include the needs for social and economic development, the needs deriving from the political and administrative institutions of society, the needs of the general cultural activity and achievements of society and the needs deriving from values and ideology. These needs, whatever they are, may not be agreed upon, but what matters is that the decision-makers, who outline the framework of mathematics education, should have these needs and priorities in mind, whatever their positions and backgrounds might be.

The second type of purposes has regard for the needs of students as individuals. These are the needs which the decision-makers on mathematics education consider useful to the individual citizen in mastering his own social and private life. They may include the following: the need to comprehend and to judge phenomena, as well as statements made about them, which occur in the natural and social environment of man; the needs associated with action and creativity; the need for emotional and aesthetic experience; and the need for attractive surroundings in one's private life.

The classification of purposes outlined above may be summarized as in the diagram opposite.

We do not claim that the above classification covers all of the purposes of mathematics education. We have, however, found that many of the goals usually put forward are based on one, or both, of the two types identified above. Later in this paper, we shall examine more closely a representative selection of goals and purposes to be found in literature on mathematics education. Meanwhile, it is perhaps relevant to remark that the two types of purposes are not necessarily independent. It will often be the case that the purpose of meeting an individual need is perfectly compatible with a purpose of meeting a societal need. For example, the purpose of providing society with well-qualified and highly specialized labour corresponds often with the purpose of providing attractive living conditions. Conversely, however, it might happen that differences of interests within a system can lead to incompatible or even contradictory purposes. It might also happen that different levels in the system (for instance, central curriculum planners on the one hand and teachers on the other) will emphasize different purposes.

	Examples of such needs
1 Purposes deriving from the **NEEDS OF SOCIETY AS A WHOLE**	(a) needs for social and economic development; (b) needs of the political and administrative government of society; (c) needs for general cultural activity and achievements; (d) needs of values and ideology.
2 Purposes deriving from the **NEEDS OF INDIVIDUALS IN MASTERING THEIR OWN LIVES**	(a) needs to comprehend and judge phenomena, as well as statements about them, which occur in the natural and social environment of man; (b) needs of active and critical participation in the democratic process; (c) needs of action and creativity; (d) needs for emotional and aesthetical experiences; (e) needs for pleasant surroundings to one's private life.

Figure 1

Pursuit of Purposes

From time to time, the teaching of mathematics becomes the subject of intense debate, not only among the pundits, but also among those in the community at large. Why is this so? Why haven't the issues been settled long ago? There would appear to be two main reasons. In the first place, as has been said, the purposes of mathematics education have always been rather obscure and more implicit than explicit. Secondly, even when purposes are established, there is no firm foundation, in terms of reliable knowledge, on the basis of which it can be judged if this way of organizing mathematics education or that will secure the purposes. This leaves us with some rather important questions unanswered. Here is such a question: 'On what grounds can we claim that mathematics will so sharpen the intellectual capacities of students as to render them capable of dealing effecitvely with situations outside mathematics?' Here is another: 'Where do we find evidence for the assertion that teaching mathematics (as distinct from teaching arithmetic and some very elementary mathematics) to masses of students is necessary for the economic development of society? Wouldn't it be more reasonable to concentrate available resources upon those students who will need mathematics in order to pursue their future professions?'

The situation would seem to suggest that, whatever view one holds on the importance of education in mathematics, that view must be founded to some extent upon very general, 'transcendental' considerations, assumptions and beliefs. As to those purposes which derive from the needs of social and economic development, the problem is of a twofold nature. First of all, the role and function of mathematics as such in social and economic development are far from well illuminated. The involvement is complicated. Its effect, most often, is the result of 'action at a distance' (in space and time). So it is not surprising that it has been difficult to determine the mathematical qualifications citizens require to participate in the processes of social and economic development. The uncertainties have presented curriculum planners with substantial problems which have led to the adoption of two contrasting strategies for meeting the needs at issue.

Strategy I may be phrased as follows. There are strong reasons to believe that a general competence in mathematics is of vital importance to the development of any society quite apart from the presence of a high level of mathematical competence among various categories of citizens. However, only very little of substance is known about the channels and paths of influence of mathematics on society. So, instead of founding our strategy for mathematics education on misleading, or even erroneous, ideas, and risking the possibility of causing serious damage (with far-reaching consequences), we should let the experts, the mathematicians and the mathematics teachers, decide from purely mathematical and didactical considerations what mathematics should be taught and how it should be taught. In short, the external restrictions placed on mathematics education should be as few as possible in order to set free the optimum of mathematical resources available in society.

Strategy II can be phrased as follows. Even if it may very well be true that not very much is known about the ways in which mathematics influences society and its development, we cannot leave mathematics education exclusively to the mathematicians and the mathematics teachers. Many examples can be given which show that such a strategy tends to produce persons whose mathematical knowledge remains encapsulated from the problems of the real world, and persons who have serious difficulty in bringing their mathematical knowledge to bear upon the problems that occur in other sciences, in technology, in the economy, and so forth. Therefore, and because manpower and economic resources are scarce, we have to formulate priorities and curriculum plans to fulfil them, plans which will ensure enough properly qualified students to further professional education. Within this strategy, the role of mathematicians and mathematics teachers is to fill the centrally fixed frames with specific content and teaching. The central examinations will ensure that the students meet the required standards.

Objectives

Although a distinction between 'purposes' and 'objectives' is normally not found in writings on mathematics education, there seems to be a tendency to concentrate upon those goals which, in our terminology, would be called 'objectives' rather than 'purposes'.

Another distinction is, however, frequently made. It is the division of the goals of mathematics education into two classes, the 'utilitarian' goals and the 'formative' goals. It seems that this distinction is not a very precise nor a very happy one to make. This is because the attributes 'utilitarian' and 'formative', as used in the debate, are far from being mutually exclusive. Goals, said to be utilitarian, are sometimes of a formative character. And goals, referred to as formative, are often backed up by utilitarian reasons. With this in mind, we shall introduce a distinction between 'formative objectives' and 'substance objectives'.

By a 'formative objective' we mean any objective oriented towards influencing the student's general personal characteristics and capacities. These would include his attitudes, his general mental capacities, his ability to perform certain general activities, etc. To these ends mathematics is merely a vehicle. The subject matter is not of itself important. The criterion for including a particular item is simply its capability of serving these formative aims. In principle, such formative objectives might be pursued using a subject other than mathematics as the vehicle. In the sense of its use here, the concept 'formative' carries its broadest meaning. It includes anything to do with developing a student's general personal characteristics.

By 'substance objectives' we mean any objective orientated towards providing a student with those competences for which mathematics as such is indispensable. The phrase 'mathematics as such' is not confined in its meaning to specific content or operations. It includes anything involving mathematical substance in one form or another.

We can now set out the relationship between purposes and objectives in this way:

purposes objectives	purposes of type 1 (social)	purposes of type 2 (individual)
formative objectives	'Having the fulfilment of the *general social need* X as a purpose of mathematics education, the *formative objective* to develop in students the personal characteristic x is put as an objective of mathematics education.'	'Having the fulfilment of the *need Y of individuals* as a purpose of mathematics education, the *formative objective* to develop in students the personal characteristic y is put as an objective of mathematics education.'
substance objectives	'Having the fulfilment of the *general social need* Z as a purpose of mathematics education, the *substance objective* of developing in students the competence z involving mathematics substance is put as an objective of mathematics education.'	'Having the fulfilment of the *need W of individuals* as a purpose of mathematics education, the *substance objective* of developing in students the competence w involving mathematics substance is put as an objective of mathematics education.'

Figure 2

A given objective may serve both a type 1 (social) purpose as well as a type 2 (individual) purpose. (See Figure 1 on page 0). For instance, the formative objective 'to provide students with the capacity of performing abstraction processes' may serve the general social demand for a qualified labour force, which, in a rapidly changing society, should be able to adapt flexibly to new situations (1a, b of Figure 1), or to the need of students as individuals to create attractive surroundings to their private lives (2e of Figure 1). It may serve also a general ideological desire that citizens should be able to identify the core of important and complicated social issues, and take positions on them (1d – and b – of Figure 1). Or it may serve the needs of students as individuals to participate actively and critically in democratic processes (2b of Figure 1). So, one and the same objective may be chosen with a view to serving simultaneously several purposes, even of different categories. Of course, this need not be the case. We can think of many objectives which serve only one purpose.

There are implications of this for analyses of the impact of social needs on the goals of mathematics education. One is that it is difficult to detect from a mere indication of objectives, as set down in official documents or elsewhere, the ultimate social interests which the stated objectives serve. In this respect, one is tempted to put forward the hypothesis that the more compound, heterogeneous and complex the interests and the political organization of a given society are, the farther away from the surface and from the daylight are the purposes of mathematics education to be found.

Elements in the Historical Development of Goals

It seems to be an established fact that, in most countries, the post-elementary education in mathematics up to the beginning of the 20th century had as its foremost goal the development of mental discipline. Geometry, in particular, was used to fulfil this task. Society needed people to occupy leading positions in politics, public administration, industry and science. For these, certain general mental capacities were desired. They included loyalty to given conditions, the ability to distinguish essentials from casual details, the capability of analysing situations, judgement, care, precision and diligence in work and an appreciation of the cultural and moral values of society. So, in this context, we are dealing with purposes of type 1. (See Figure 1 on page 7). But, of course, mental discipline was also to some extent considered useful to the individual in mastering his own life (purposes of type 2). The degree of emphasis on purposes of types 1 and 2 undoubtedly differed very much from country to country. As examples, Germany might, perhaps, be cited as one which favoured purposes of type 1, and England and Wales, those of type 2.

Only a small minority of the population was educated in mathematics beyond the level of elementary arithmetic. The instruction given to the majority in elementary arithmetic aimed primarily at providing pupils with certain computational skills. These were deemed to be useful for them in their adolescent lives (purposes of type 2), as well as being demanded by society in general (purposes of type 1). But, with an increasing emphasis on the latter as industrialization grew, the instruction was aimed also at developing mental discipline. It may be supposed that the purpose of this was to serve the increasing demand for a careful, precise, diligent and loyal labour force, particularly for the growing industry.

The idea of mental discipline stemmed from faculty psychology. According to this, man possesses certain general, context-free faculties. These, it is alleged, can be developed in any context and can be freely transferred to any other context. In accordance with these ideas, the purposes of post-elementary mathematics education, whether dominantly of type 1 or of type 2, were pursued via objectives of a formative character.

Around the turn of the century, the changes in the socio-economic and technological conditions of many countries, led by those of Western and Northern Europe and the United States of America, brought about manor changes in the educational needs of these countries. In mathematics education, this development let to an increasing emphasis on 'the use' of mathematics in society. In *A History of Mathematics Education in the United States and Canada*[1], De Vault and Weaver characterize the mathematics for grades kindergarten to 6 during the period 1894–1923 in the United States as follows: '. . . we teach mathematics because adults use mathematics in their everyday work requirements.' In the same Yearbook, Osborne and Crosswhite refer to observations made on grades 7 to 12 around 1912–17 by an American committee under the International Congress of Mathematicians. These were that utilitarian aims were becoming increasingly important, and that the concept of education as a formal discipline was being questioned.

The trends were similar in Germany. In 1915, the Association for the Promotion of Mathematics and Science Instruction ('Der Verein zur Förderung des mathematischen und naturwissenschaftlichen Unterrichts') arranged a prize competition on the question 'What demands should be made on the education of the German youth after the war, and what can mathematics and science instruction contribute to the realization of these demands?'[2] The first prize was awarded to Dr. W.

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[1] *Thirty-second Yearbook of the National Council of Teachers of Mathematics.* Washington D.C., 1970.
References to:
Part Two: "Forces and Issues Related to Curriculum and Instruction, K–6", by M. Vere De Vault and J. Fred Weaver, p. 110;
Part Three: "Forces and Issues Related to Curriculum Instruction, 7–12", by Alan R. Osborne and F. Joe Crosswhite, p. 183.

[2] Von W. Schmiedeberg et al. *Die Bedeutung des mathematischen und naturwissenschaftlichen Unterrichts für die Erziehung unserer Jugend,* Preisschriften des Vereins zur Förderung des mathematischen und naturwissenschaftlichen Unterrichts. Verlag Otto Salle, Berlin, 1917.

Schmiedeberg, master at 'der Oberrealschule zu Bielefeld'. His contribution possess explicitness, thoroughness and clarity to a degree which is indeed rare. Schmiedeberg points to four general purposes of education in school: education for national defence; education for serious, diligent and conscientious labour; education for working in a community; and education for patriotism. He summarizes these purposes as follows: 'We need strong people who can defend themselves, industrious people who can create economic greatness, loyal and patriotic people who will commit their power and labour consciously to national aims.'[3] He goes on to say: 'If we recall that the future of Germany depends on the maintenance and further development of work of quality, we realize that education conducive to absolute devotion to duty should remain our foremost educational aim for all kinds of schools.'[4] Various components in these purposes are put forward: 'education for regularity and stability', 'subordination of the individual to the organism of work', 'preparation for adaptation and obedience'. The purpose of mathematics education should be to contribute to the fulfilment of these general purposes of education (the foremost means being the applications of mathematics).

Even if Schmiedeberg's views may seem today to be rather extreme, they were evidently approved by the Association. And we think it not unjustifiable to suppose that Schmiedeberg was expressing views which were generally held at that time, at least in Germany.

Thus far, it would seem reasonable to claim that purposes of type 1 held an explicit and dominant position during the first decades of the 20th century, and that purposes of type 2 were less in evidence. In England, however, the situation was somewhat different. In 1901, John Perry, a professor of engineering science, maintained, in opposition to the mainstream of belief in England at that time, that the study of mathematics began because it was useful, continues because it is useful, and is valuable to the world because of the usefulness of its results.[5] He indicated eight 'obvious forms of usefulness' in the study of mathematics. They are often quoted in writings on mathematics education:

'(1) In producing the higher emotions and giving mental pleasure.

(2) (a) In brain development.
    (b) In producing logical ways of thinking.

(3) In the aid given by mathematical weapons in the study of physical science.

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[3] *Ibid.*, p. 5, translation.

[4] *Ibid.*, p. 14, translation.

[5] *Teaching Mathematics in Secondary School.* Ministry of Education Pamphlet 26, Her Majesty's Stationery Office, London, 1958.

(4) In passing examinations.

(5) In giving men mental tools as easy to use as their legs and arms, enabling them to go on with their education (development of their souls and brains) throughout their lives, utilizing for this purpose all their experience. This is exactly analogous with the power to educate one's self through the fondness of reading.

(6) Perhaps included in (5): In teaching a man the importance of thinking things out for himself and so delivering him from the present dreadful yoke of authority, and convincing him, that, whether he obeys or commands orders, he is one of the highest beings.

(7) In making men in any profession of applied science feel that they know the principles on which it is founded and according to which it is being developed.

(8) In giving to acute philosophical minds a logical counsel of perfection altogether charming and satisfying, and so preventing their attempting to develop any philosophical subject from the purely abstract point of view, because the absurdity of such an attempt has become obvious.'

These statements have often been interpreted as being in keeping with the general stream of social-utilitarianism of that period. However, on closer analysis[6], this interpretation becomes doubtful. Most of Perry's 'forms of usefulness' refer directly or indirectly to the needs and interests of the individual (type 2 purposes). The possible exceptions are (2) and (7). These could be considered as referring also to some form of social utility (purposes of type 1). As to (3), we are not able to tell whether it refers to purposes of types 1 or 2.

As to the type of objectives which underlie Perry's forms of usefulness, we venture the conjecture that they are mainly formative in character.

Although, in the case of Perry's suggestions, we are essentially not dealing with purposes of type 1, the promotion of such purposes continued, in England, in the course of the first decades of the century. In 1919, a report of the Mathematical Association stated as follows:

'1. That a boy's educational course at school should fit him for citizenship in the broadest sense of that word: that, to this end the moral, literary, scientific (including mathematical), physical and aesthetic sides of his nature must be developed. That, in so far as Mathematics is concerned, his education should enable

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[6] Sean McNealis and Jas. A. Dunn, 'Why teach mathematics?', *Int. J. Math. Educ. Sci. Technol.*, 1977, Vol. 8, No. 2, pp. 175–84.

> him not only to apply his mathematics to practical affairs, but also to have some appreciation of those greater problems of the world, the solution of which depends on Mathematics and Science.
>
> '2. That the utilitarian aspect and application of mathematics should receive a due share of attention in the mathematical course.'

Thus, purposes of type 1 gained momentum, but purposes of type 2 were still important:

> 'We assume that the majority of boys will not make any extensive or profound use of Mathematics in after-life, that they will not even be able to follow in detail the mathematical methods of engineering and applied science. But they can be so taught that a vista is opened up to them through which they may see the tremendous potentialities of the study whose elements they are mastering. A public must be created able to realise what Science and Mathematics are doing for the world, and to form some general conception of the means employed. The average man will not be more than a spectator of the world's material progress; we have suggested that he should be an intelligent spectator. But the world needs an increasing number of workers trained in mathematics; there must be specialists.
>
> 'We do not hesitate to put the pleasure motive among the foremost.'

During the twenties and the thirties purposes of type 1 maintained and strengthened their central, and somewhere dominant, position in the industrialized world. Due to the stagnation and recession of industrial and general socio-economic development connected with the world crisis of the early thirties, there was, however, no large-scale qualitative technological expansion which required fundamentally new competences in the labour force. So, there was no external pressure for re-examining the purposes of mathematics education. This led to somewhat of a fossilization of arithmetic teaching. And mechanical routine in computation became a main object of it. Furthermore, post-elementary mathematics kept to traditional paths. The only innovations at this level were the attempts at unification by replacing the traditional compartmentalization of the curriculum with a 'mixed' approach. As to objectives, a trend of switching weight from formative to substance objectives (without dispensing with the former) in the pursuit of purposes of type 1, and type 2 as well, could be detected in several countries.

In the late thirties, certain new signals could be detected. Several observers and commentators found that too much emphasis was being laid on mechanical drill and on obtaining correct results, and that too little attention was being paid to students' understanding of the mathematical background to these routine operations. This did not mean that the purpose of social utility was being seriously questioned. De Vault and Weaver, in *A History of Mathematics Education in the United States and Canada,* quote Brueckner for having written (in 1941): 'Arithmetic should be both mathematically meaningful and socially significant.'[8] In England, the Spens Report (1938) of the Consultative Committee on Secondary Education stated:

> 'No school subject, except perhaps Classics, has suffered more than Mathematics from the tendency to stress secondary rather than primary aims, and to emphasize extraneous rather than intrinsic values. As taught in the past, it has been informed too little by general ideas, and instead of giving broad views has concentrated too much upon the kind of methods and problems that have been sometimes stigmatised as "low cunning". It is sometimes utilitarian, even crudely so, but it ignores considerable truths in which actual Mathematics subserves important activities and adventures of civilized man. It is sometimes logical, but the type and "rigour" of the logic have not been properly adjusted to the natural growth of young minds.'[9]

Because of World War II, the growing demand of mathematicians and of mathematics educators that students should know and understand the mathematical background of what they were doing did not come immediately into effect. After the war, socio-economic and technological conditions were totally different from those which preceeded it. All over the world, the restoration of industrial production from a war-time to a peace-time basis, and the associated needs for economic growth, influenced requirements, and confronted the educational sector, including mathematics education, with a new challenge. Technology was no longer static. It expanded qualitatively as well as quantitatively, with the United States in the front of the wave. One result of this expansion was a demand for a well educated labour force capable, at all levels, not only of performing routine operations, but also of adapting to more rapid changes in technology and working conditions. As De Vault and Weaver put it, when commenting on the 'Sputnik shock'. 'For both the average citizen, and the worker employed in the expanding economy, mathematics was essential. Increasingly, it was recognized that the man on the street needed to be knowledgeable in mathematics if he was to understand the world in which he lived.'[10]

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[8] Leo J. Brueckner. "The Social Phase of Arithmetic Instruction" In: *Arithmetic and General Education, Sixteenth Yearbook of the National Council of Teachers of Mathematics.* New York, Bureau of Publications, Teachers College, Columbia University, 1941. p. 121.

[9] *Teaching Mathematics in Secondary School.* Ministry of Education Pamphlet 26, Her Majesty's Stationery Office, London, 1958, p. 17.

The situation described gave rise to an alliance between mathematicians, mathematics educators and educationalists. This was concluded by university mathematicians, on the one hand, and society (represented by government institutions and international organizations) on the other. Society sought a system of mathematics education which would first of all serve purposes of type 1, and second of type 2. The communities of mathematicians and mathematics educators offered to serve these purposes with thorough renewal of mathematics curricula at all levels. It may be conjectured that the post-war economic expansion was, in itself, less essential to many of these reformers than the opportunity it gave them to renovate mathematics education, and, hopefully, to stimulate the numbers and the quality of recruits to university studies in mathematics and in natural sciences.

This development constituted the so-called 'first wave' of the 'new math' reform. It was finally carried through in most countries at the beginning of the seventies. Since this development has been well-illuminated by observers and commentators, we shall not discuss it in detail. Instead, we shall comment upon certain recent trends.

During the period of consolidation after the culmination of the 'first wave', various new traits have emerged. They originated from the observations that:

(1) a strikingly large proportion of students of all grades had serious difficulties in grasping the rather abstract and structure-oriented content, which had become common as a result of the 'first wave';

(2) the new curricula did not come up to the expectations and promises that they would enable students to use mathematics in non-mathematical contexts with intelligence and flexibility. This led to complaints by institutions and employers;

(3) students became increasingly critical about being forced to learn difficult subject matter, the relevance of which was not only obscure to them, but also was being questioned by outside observers.

As a result, mathematics educators began to reconsider the goals and curricula of mathematics education. Bent Christiansen identifies[11] the following directions of change in goals:

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[10] *A History of Mathematics Education in the United States and Canada*, p. 135.

[11] Bent Christiansen. *European Mathematics Education – The Past and Present.* Royal Danish School of Educational Studies, Copenhagen, December 1975, p. 10.

'2.6.1 An increased emphasis on affective goals.

'2.6.2 An increased acceptance of responsibility for the majority of students, not only for an élite.

'2.6.3 An increased emphasis on usefulness (in an ordinary daily life sense) and on applicability of the outcomes of mathematics teaching.

'2.6.4 An increased concern for the individual learner; for learning more than for teaching; for the learner's experiences, needs and interests, more than for the requests of mathematics as a science.

'2.6.5 An increased acceptance of the role of the mathematical activities (involved in processes such as description, systemization, problem-solving, proving) . . . . not only knowledge of the results of mathematical activities, but also experiences from a familiarity with the processes that lead to the results are seen as main objectives.'

Z. P. Dienes suggests[12] 'that the main aim of mathematics education, instead of being the knowledge of mathematics, should be the development of certain patterns of thinking, certain types of strategies, that people might develop in the face of new situations which they have never encountered before.' He finds that mathematics education may provide students with four competences, namely to abstract ('The process of getting rid of irrelevances and cutting through noise (in information theory terms), and getting down to the real message is certainly a particular competence which would be a great asset to people in the modern world.'), to generalize ('This kind of enquiry obviously has a great interest in a very fast expanding and modern world' and '. . . what would happen if . . . ?', 'supposing that . . . ?'. 'This kind of enquiry is very important in any kind of modern development, be it government, industry, or scientific research. The modern man should always consider the possibility of change. In fact he has no choice. He lives in a world of change and he has to adapt to it.'), to decode and to encode messages ('So, learning to cope with precise messages or learning to formulate precise messages would be an important side effect of having learned to cope with mathematical messages in general.'). In his conclusion[13] Dienes says:

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[12] Z. P. Dienes. "Learning Mathematics" In: G. T. Wain (ed.), *Mathematical Education*, chapter 6. Van Nostrand Reinhold Company, Wokingham (England) and New York, 1978; p. 82-3.

[13] *Ibid.*, p. 94.

> 'To sum up, the suggestion is being put forward that the learning of mathematics could and should have other pay-offs besides the the knowledge of mathematics . . . . Last, but certainly not least, is the social pay-off. Children learn to work together in collaboration instead of in competition with one another while discussing problems such as have been described. They learn to respect the opinions and values of others and thus develop their own opinions and values within a practical and realistic social context.'

Using these few references to indicate the latest trends in goals of mathematics education, (several other references could be given), how might these trends be interpreted within our framework for analysis?

As to purposes, the direction taken after the first wave of reform is towards placing increasing importance upon the needs of individuals, first as students, later as adults. These are purposes of type 2. Particular attention is being paid to those needs which are associated with living in, understanding and participating in a society of a rapidly growing complexity, characterized by alienation and by narrow ranges of possible courses of action, originating from, among other things, optimization and control of the exploitation and allocation of scarce resources. One could perhaps say that the increasing stress upon purposes of type 2 reflects an attempt to provide individuals with a means of protecting themselves against the consequences of certain lines of development in society.

The needs of society as a whole – the purposes of type 1 – still have a central position in mathematics education. Up to the seventies, society required manpower which could take part in the technological and economic expansion, which accompanied an extension of the apparatus of production and of the public sector, and by submitting ever new areas of activity to technological treatment or innovation. During the seventies, and concurrently with the emergence of the world economic recession, the economy switched from an extensive to an intensive exploitation of capacity, with automatization, optimization and rationalization as important ingredients. This development led to a decreasing demand of labour. A result of this development is an increasing need for an adaptable labour force, with general rather than specific competences. For mathematics, this implies a capability of using mathematical tools in a variety of contexts. So, in terms of objectives, there has been a change in stress, after the 'first wave' of reform, from substance objectives to formative objectives. The stress now is on such capacities as: abstraction, generalization, creativity, general problem-solving behaviour, co-operative behaviour, participation in collective tasks, decoding and encoding of messages, transfer and the like. Also the character of substantive objectives has changed.

While, in the beginning of the reform period, substance objectives stressed concepts which were internal to mathematics, they are now more concerned with mathematics in relation to its surroundings and, in particular, the skills of model-building and problem-solving.

The objectives used to pursue purposes of type 2 are partly the same formative and substance ones needed for purposes of type 1. Some additional formative objectives, frequently suggested, seem, however, to refer particularly, even if not solely, to purposes of type 2: the fostering of curiosity, attitudes of research, critical attitudes, capacity to argue, emotional and aesthetical joys and the like. This trend towards emphasizing the needs of individuals is supported by a parallel trend lately of growing concern about respect for the integrity of children, discovery, intuition, play and experiment.

We conclude this section by summarizing the major trends of change in the goals of mathematics education during the periods treated as follows:

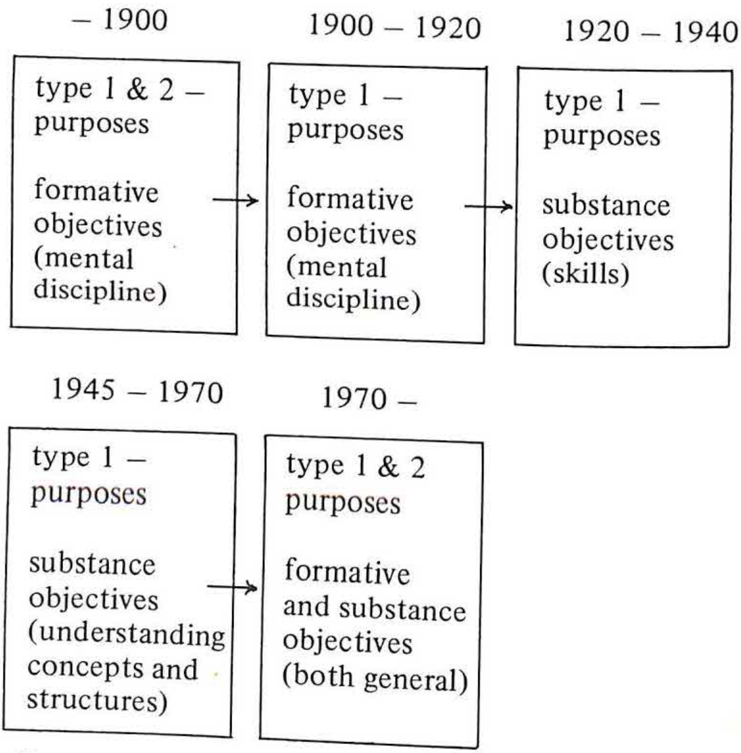
– 1900
type 1 & 2 – purposes
formative objectives (mental discipline)
1900 – 1920
type 1 – purposes
formative objectives (mental discipline)
1920 – 1940
type 1 – purposes
substance objectives (skills)
1945 – 1970
type 1 – purposes
substance objectives (understanding concepts and structures)
1970 –
type 1 & 2 purposes
formative and substance objectives (both general)

Figure 3

Even though this historical sketch identifies only the main trends, and does this by referring to singular cases, we think it is fair to claim that society has always held that the main purposes of mathematics education are to serve the needs of society as a whole – what we have called purposes of type 1. However, purposes of type 2 have occupied positions of varying strength as economic, political and ideological conditions have fluctuated, and as humanistic ideas have had influence. Even so, it seems that society has always considered these purposes to be subordinate in principle to purposes of type 1.

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Margaret Brown

Goals as a Reflection of the Needs of the Learner

Introduction

During the economic boom of the nineteen fifties and sixties, in almost every country in the world, committees were busy planning widescale improvements to the mathematics curriculum. These committees contained mathematicians, mathematics educators, administrators and mathematics teachers, in varying proportions according to the political and educational ideology of the individual country.

But, in almost all cases, the concern over the teaching of mathematics was restricted to those working within the area of mathematics. The objectives, in general, were to increase the supply and quality of mathematicians and mathematics teachers, and to give all children an exciting and up-to-date picture of what mathematics was about. This included in some cases, how it was used, and, in others, what it felt like to be creative in mathematics. The revolution, in other words, was rooted firmly in the needs of mathematics regarded essentially as a discipline.

Unfortunately, it would seem, the main effect of all the curriculum changes was the growth of a sense of disillusion in the nineteen seventies. Not only were there few indications that mathematical standards were any higher as a result of the new curricula, there were even complaints of lower standards, some from within[1], and many from without, especially from associated disciplines like science and engineering. What was even more strange were the occasional indications that children doing 'modern mathematics' showed less liking for, and less commitment to the subject than previously[2].

[1] See, for example, Thom, R. 'Modern Mathematics: Does it Exist?' In: Howson, A.G. (Ed.) *Developments in Mathematics.* Cambridge, Cambridge University Press, 1973; and Kline, M. *Why Johnny Can't Add.* New York, St. Martin's Press, 1973.

[2] See Preston, M.H. *The Measurement of Affective Behaviours in CSE Mathematics,* unpublished Ph.D. Thesis, University of Leicester, 1973; and Kempa, R.F.; McGough, J.M. A Study of Attitudes Towards Mathematics in Relation to Selected Student Characteristics. *Br. J. Ed. Psych.*, Vol. 47, pp. 294–304, 1977.

So, what went wrong with the visions of the sixties? Today, other committees sit pondering over the mathematics curriculum. But now, in the Western world, they do so against a background of economic recession and growing unemployment. Today, the committees are more likely to contain, in addition to mathematical interests, those who are concerned with the employment of new recruits at all levels of educated manpower. They may also include a few token scientists or engineers, and even the odd parent or two. The objectives this time are related more to society. So they will focus on the needs for skilled labour, and on those for numerate citizens able to make informed decisions on behalf of themselves, their families and society in general. They will even focus on the 'minimal competencies' required for social survival and the avoidance of dependency on state support.

Changes in the curriculum are being recommended. Some have already been implemented. These are likely to be more realistic and less optimistic than in the previous round of curriculum change. They will probably be less widescale. But have we any guarantee that they will be more effective?

The tendency of reforming committees both now and in the past has been to concentrate upon the endpoint: what does the child need to know when he leaves school; does it serve the needs of 'mathematics-as-a-discipline' or does it serve the needs of society? From such decisions one can work back, deciding on the prerequisites (e.g. if he needs to understand percentages, then, sometime earlier, he must have learnt about multiplication), and thus dividing the curriculum into neat, logically organized packages to be fed to children age-group by age-group, so that, at the end, they will emerge knowing just what is required.

The flaw in this standard approach is the risk of losing sight of the one for whom all the planning is done, namely the learner. Particularly, the learner as he is now, in the classroom at 11.27 on a Friday morning, rather than how he might be in five years time. The curriculum is not intellectual food to be fed by the teacher to the child in chunks, according to the edict from on high. At the point of transfer, a complex interaction is in process between a mathematical idea, an individual child with his present personal perceptions, interests and capabilities, and his teacher, who equally has his particular personal perceptions, interests and capabilities.

The needs of the learner as perceived by the 'committee on high' may thus be very different from the view as seen from the reality of the classroom.

What are the long-term needs of the learner?

A *general* view of the long-term needs of the learner would probably run something like this: (a) the ability to fulfil satisfactorily his role as a member of a variety of different social groups; (b) the enrichment of all aspects of his own personal life as an individual.

The first of these clearly relates to the needs of society, and stresses the mutual dependence between the individual and the group. The second, perhaps less obviously, relates to the needs of mathematics as a discipline. In order to enrich the experience of the individual one would expect to enable him at least (i) to master as much actual mathematics as he can; (ii) to understand the nature of mathematics, including how it is created and how it is used; (iii) to appreciate aesthetically mathematical symmetries; and (iv) to enjoy mathematical problem-solving. All these would inevitably serve the needs of the subject in addition to those of the individual.

Like the twin major pillars of equality and freedom, the problems only start when we are required to weigh one against the other in a practical decision. It may then be an invidious task for an administrator or curriculum planner to decide whether the social or the individual aspect takes precedence. For instance, should one encourage a student to study for a university degree in pure mathematics when the only jobs available are for factory-hands? Or should one encourage low attaining fifteen-year-olds to take courses in 'social arithmetic', which include no element of the appreciation of mathematical structure? And what about the rumoured senior ministry official who rejected modern mathematics on these grounds: 'But we don't want them to think, and especially not the girls'!

We can all agree about a list of desirable aims; the difficult bit comes only when two such aims are in conflict.

Varying perceptions of long-term needs

Conflict is also likely to arise between the 'official' view of the needs of an individual and the perceptions of that individual himself.

Even the child of eleven has, in general, realized that mathematics is important because it can help him in the future, both in obtaining and in succeeding at a job, as well as in his everyday life[3]. Indeed we know that the role of education in preparation for a job is seen by the child and his parents as much more important than it is by his teachers[4]. And both are likely to perceive the importance of passing examinations in order to qualify for particular jobs or training.

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[3] See, for example, Assessment of Performance Unit (APU). *Mathematical Development: Primary Survey*. London, Her Majesty's Stationery Office *for* the Department of Education and Science, 1980.

In general, therefore, the socio-economic objectives of the 'committees of the nineteen seventies' are likely to be shared by the child, the parents and the teacher.

However, there are bound to be particular instances where there is conflict.

It is by no means easy for the teacher to persuade a boy who might be leaving school the following year to take up a well-paid job in printing, as his father did, that he really ought to learn trigonometry because it won't be too long before he's made redundant and he may then need this knowledge. (It is made even more difficult when both teacher and student are aware of the fact that he's likely to have forgotten trigonometry by then anyway!).

And, even if the long-term needs are perceived by the pupils, there may be a very different perception of the means of attaining them.

I was recently in a girls' school where the mathematics syllabus for the very weak fifteen- and sixteen-year-old girls was divided into three sections: 'computation', 'general mathematics' and 'social mathematics', each of which was taught on a different day of the week. The 'computation' was a series of standard, pencil-and-paper algorithms of a type that most educators would agree was obsolete in a calculator age. The 'general mathematics' was an individualized programme, aiming at the understanding of basic concepts like area, decimals, and so on; this was presented in a fairly typical 'modern-type' pedagogy using concrete apparatus, including calculators. The 'social mathematics' was based on a lavishly illustrated textbook which developed various realistic situations in the lives of two young people; it attempted to involve the pupils in the necessary decision-making.

When asked about the various strands of their curriculum, the girls found it very difficult to express their views articulately. But they were in general agreement that the 'computation', although boring, was 'very important'; the 'general mathematics' was 'not very important', although 'quite interesting'; and the 'social mathematics' was 'useless' and 'boring'. When pressed, the latter judgment seemed to be based partly on the fact that the text had too many words, partly on the perception of the subject-matter as 'low status knowledge', and partly on the failure of the girls to identify themselves with the characters in the way that had been intended.

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[4] See, for example, Schools Council. Enquiry 1: *Young School Leavers.* London, Her Majesty's Stationery Office, 1968.

However, when we turn to the 'needs related to the enrichment of the life of the individual', which, as we have seen, correspond to the 'discipline-based' needs, we may be treading on much more thorny ground.

Here, the 'committee of the nineteen sixties' may have included some elementary group theory in the curriculum with two aims in mind: one of giving the individual some insight into the structural nature of mathematics, the other of increasing his aesthetic appreciation of its symmetries. The child, on the other hand (and, perhaps, with the active connivance of his teacher), may regard this topic as just one more set of words and techniques to be learnt in order that they can accurately be reproduced at examination time. Whether he is a teenager from an urban housing estate facing the prospect of unemployment, or, indeed, whether he is a son of bourgeois parents eager to push him into a 'respectable' career like accountancy, there is a strong chance that he will find elementary group theory irrelevant to his long-term needs, either as an individual or as a future worker. His family background, however deprived or affluent, may be such as to encourage him to perceive only the instrumental, job-related role of education, and to have little appreciation of the liberal view of education as a fulfilling, mind-broading experience.

Whose views should prevail? On the one hand, no responsible teacher would always bow to the opinion of his pupils, and, as in the case of the teenage girls referred to earlier, prescribe a diet restricted to routine computation just because the girls thought it to be the only thing that was 'important'. On the other hand, if the divergence between the curriculum of the committee and the learner's own perceptions of its value is too great, the child may simply 'vote with his feet', and, even if he does not physically absent himself from the classroom, he will opt for a policy either of 'maximal diversion' or of 'minimal engagement' or of both. The children whose behaviour causes problems as a result of their disenchantment with what they are offered are the most noticeable ones, but many times more numerous than these are those who enter a silent pact with their teachers not to cause disruption provided no more than minimal participation is required – a few 'sums' here, a 'drawing' there, but nothing really to divert the mind from its ever present preoccupation with teenage culture.

Any resolution, therefore, must be one of constructive compromise – just as it would be if a divergence of views occurred between two individuals engaged in a common pursuit. First, there needs to be communication: each must take the trouble both to inform the other of his position and of his justification for it; each must listen to the case put by the other side. The negotiation of an agreed position may then start, provided that each side respects the position of the other and the right of both sides to participate. Arrogance may result temporarily in the winning of the argument, but only at the cost of long term resentment, future lack of commitment and possibly complete breakdown of the relationship.

In education, the situation is not a symmetric one: there are those who have a right to impose a curriculum, but there are also those are able, in practice, to reject it, with the teacher caught in an unhappy position between the main parties. In order that the educational enterprise should be successful, it is necessary that the 'committee-on-high' should communicate their objectives to pupil, to teacher and to parent; they, in turn, should actively seek to determine the views of all three groups. The more active the participation of teachers and pupils in the decision-making, the greater will be their commitment to the teaching and learning of the curriculum in the spirit intended.

What are the short-term needs of the learner?

In order to engage fully in a learning task, the child in the classroom needs to be convinced that the task is either: (a) worthwhile, in that it is, or will come in useful; or (b) pleasurable; but, preferably, that it is both.

The child's perceptions of what is *worthwhile* for his future needs have already been discussed. Motivation on this score may be strong in the final years of schooling, but, in general, the child's current needs are likely to prove a much more powerful incentive to learning. For example, the six-year-old wants to be able to 'tell the time' *now*, or, at the latest, by next week. The fourteen-year-old needs to know some trigonometry for *tomorrow's* science lesson. And children of all ages are consumers, who need continually to be able to make sensible decisions, even if only on a small scale. Curriculum developers might, with advantage, focus more attention on the child's perceived needs as he is now, rather than those of the adult he will sometime become.

Pleasure can arise in various ways, in the learning of mathematics. For example: (i) enjoyment of social co-operation and of social approval; (ii) aesthetic appreciation, whether intellectual or sensory in nature; (iii) satisfaction in the solution of a problem.

Let us look, for a moment, at each of these.

Social pleasure

This form of need is rarely taken seriously by designers of mathematical curricula, although it is frequently used by the teacher as a means of motivation. Not only is the teacher able to bestow his own praise, he may also choose to ask children to present their work for public approval, either by giving a short oral description of it to the rest of the class, or by displaying it on a wall. This can create pleasure by enhancement of the child's self-image. It can also serve the important aim of encouraging children to communicate clearly about mathematics, whether in writing or speech.

Both confidence and the skills of communication can also be improved by enabling children to work co-operatively. However, the major cognitive gain from group-work may be that, in arguing out his ideas with others, the child is placed in a position where he is more likely to resolve the contradictions inherent in his own reasoning.

Hence the curriculum planner and the teacher can capitalize on the child's need for social approval and social co-operation to satisfy his cognitive, as well as his affective, needs.

Aesthetic pleasure

Just as we hope that children will appreciate Beethoven or Picasso both through the mind and the senses, we should likewise give children the opportunity to admire beautiful mathematics, whether in the form of visual or symbolic symmetries, or in the more intellectual form of elegant reasoning. However, enjoyment cannot be imposed. If students generally remain indifferent to a piece of mathematics which was included in the curriculum mainly on the grounds of its aesthetic attraction, the piece should, like Euclid, be abandoned forthwith.

Creation of beauty is probably more satisfying than the mere contemplation of it. Opportunities for this can be made at all levels, from the creative drawing of symmetries and the production of number patterns right up to the construction of proofs.

Pleasure at solving problems

Perhaps the unique contribution of mathematics to the curriculum is the opportunity it offers for the emotional and intellectual satisfaction which can result from successful problem-solving, even where the problem is only that of understanding something which was previously obscure. However, the joy seems to be proportional to the cognitive struggle involved. Although success in a trivial workcard task of the 'copy and complete . . . ' variety will give some degree of satisfaction, such a task is likely to offer little frustration, to involve little commitment of effort, and hence to lead to little learning.

At the other extreme is the challenging problem which would lead to significant changes in the cognitive structure of the learner and give great joy if it were solved, but which offers correspondingly greater chances of failure and frustration. Frequent experiences of failure in problem-solving are likely to lead to a 'MAXIMIN' strategy on the part of the student – one of playing safe and accepting only the smallest degree of cognitive strain. Thus, to many children, especially at secondary level, the joy of solving problems is denied by their own or their teachers' refusal to undertake anything but the most mathematically trivial of tasks. This preference may indeed be encouraged by the increasing trend towards the individualization of learning. This often leads to the curriculum being split into thousands of individual worksheets, each demanding so simple a step as will avoid the child requiring the help of the teacher.

The only way to reverse this tendency would seem to be to produce challenging problems at the level of each learner or group of learners, which would permit a reasonable chance of success, resulting not only in increased satisfaction, but in significant learning.

Do present mathematics syllabuses reflect what we know about how children learn?

In the previous section it has been argued that a fine balance must be struck between the cognitive stage reached by the learner and the problems presented to him. Work that is too easy leads to little learning and minimal pleasure, whereas work that is too hard leads to continual failure and subsequent lack of commitment. This seems obvious enough. But, in fact, there is very little history of detailed diagnosis and assessment in the area of mathematics education.

We have tended, rather, to throw new work at the children without trying to establish what they already understand – 'because they are 12 years old they learned the addition of fractions last year, and they will learn how to multiply fractions next week'. It is just unfortunate if one half of them do not understand what a fraction means or for what purpose it is used.

The armchair educator can reason that the concept of multiplication must precede that of percentage. But he can neither deduce how much easier the former is than the latter, nor what can be learned between both, nor to what ages or abilities of children either concept can effectively be introduced.

In the light of this remark, the remainder of this section is devoted to the reporting of some of the findings of the 'Concepts in Secondary

Mathematics and Science (CSMS)' programme based at Chelsea College, University of London between 1974–79, with which the author has been associated.

The aim of the project was to reduce the mismatch between the learner and the curriculum, by (i) identifying the order of difficulty of the individual topics normally found in mathematics courses, thus producing a 'concept map'; (ii) pinpointing particular learning difficulties; and (iii) providing evaluatory procedures designed to help teachers to identify the stages reached in their pupils' thinking.

In their attempt to realize this aim, the research team studied the following eleven topics in mathematics: Number operations, Ratio, Fractions, Place value and decimals, Positive and negative numbers, Measurement, Algebra (generalized arithmetic), Graphs, Vectors, Matrices, and Reflection and rotation.

By interviewing more than 300 children, and by giving pencil-and-paper tests to more than 10,000 others, we were able to arrive at a series of 'levels of understanding' in each topic, and to relate these levels to some extent across the topics.

The particular abilities which seemed to characterize the children at each 'level of understanding' in each topic have been described in general terms, and they have been illustrated by reference to the type of questions those children would be able to answer (and those they would not).

In addition, the proportion of English children at each of the levels in each topic was estimated for each age-group in the secondary school.

The research instruments which were employed have now been adapted for use by teachers so that they can determine the 'levels of understanding' of children in their classes, and hence plan appropriate work in each topic to consolidate the child's present 'level of understanding' and to lead the child on to the next level. (These tests[5] are currently being published by the National Foundation for Educational Research, Slough).

The detailed results of this work occupy a full-length book[6], and a long research monograph[7]. Accordingly, only some of the major findings are summarized below.

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[5] Brown, M. Teachers' Guide to the CSMS Number Operations Test, Slough, National Foundation for Educational Research, 1979.

[6] Hart, K.M. (Ed.) *Children's Understanding of Mathematics, 11–16.* London, John Murray, 1980.

[7] Hart, K.M. *Secondary School Children's Understanding of Mathematics: a report of the Concepts in Secondary Mathematics and Science Programme.* London, Chelsea College, 1980.

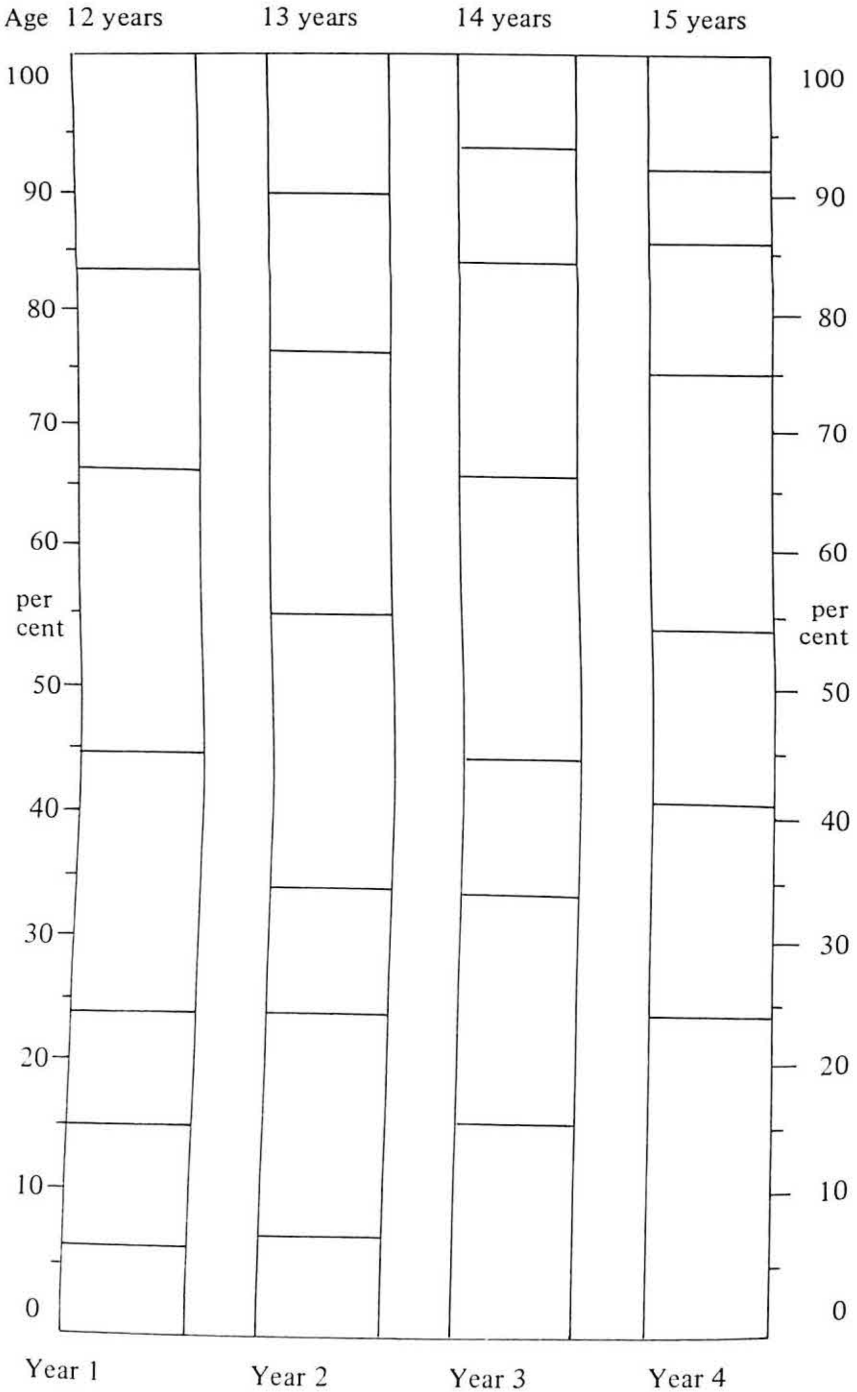

Figure 1.

Percentage of children in each year of the secondary school (ages 12–15) who are judged to have attained each level in 'place value and decimals'.

a) *There is a very wide range of understanding in each topic in each year-group in the secondary school, and there is little difference in this range from one year group to another.*

This is illustrated in Figure 1 by the results of the study of 'Place value and decimals'. In this study, seven 'levels of understanding' were identified. Figure 1 depicts the proportions of children in each age-group between 12 and 15 years at each of the seven levels. The normal distribution of ability was allowed for by matching the size of the samples to the known IQ distribution of the English population in each age-group.

Children at level 0 had little grasp of the basic ten place value system used for natural numbers, whereas those at level 6 had a comprehensive understanding of the base ten system as used in the representation of natural numbers and of rational numbers[8].

It can be seen that, although the actual proportions change from year to year, all levels of understanding are present in each of the age-groups in significant proportions (i.e. at least six per cent). This implies that a teacher of a mixed ability class in any age-group can expect to find at least two children at each of the seven levels of understanding!

The Figure also shows that, in their understanding of this topic, the top 20 per cent of 12 year-olds are ahead of the average 15 year-old. Indeed, these children would seem to have more in common with the brightest 15 year-olds than with the average 12 year-old. Conversely, the weakest 20 per cent of 15 year-olds are behind the average 12 year-old. Their degree of understanding is more akin to that of the weak 12 year-olds than to that of the average 15 year-old. These findings for this topic were not unique. A similar pattern was obtained in each of the curriculum areas chosen for analysis.

The implication of this finding is that it is clearly inappropriate to produce curriculum materials which cater for specific age-groups – as, for example, a series of textbooks with one book for each year in the school. Even if the writers of such a series were to aim at the median 'level of understanding', the materials for the 12 year-old would be 'poorly-matched' to 40 per cent of 12 year-olds, but 'well-matched' to 40 per cent of 15 year-olds! This judgement is based on the criterion of 'differing by not more than one level' as defined by our system. It is, of course, an arbitrary criterion, and it assumes a linearity of the 'levels', which does not necessarily exist. Nevertheless, the writer believes the conclusion has some validity.

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[8] See Hart, *Children's Understanding . . . op. cit.* for a more detailed definition of these levels.

b) *The progress of individual children is very slow*

Figure 1 suggests that, over the three-year period from age 12 to age 15, very few children would advance by more than two of the seven levels: about one half would grow by just two levels and about a third by just one level. However, this inference cannot be supported by other evidence, as the 12 and 15 year-old samples tested in this topic were cross-sectional, and therefore contained different children.

There is, however, some evidence in the case of three other topic areas, where a longitudinal survey *was* carried out with a sample of about a hundred children, spanning the whole of the ability range. The children were tested in each of three successive years, from age 13 to 15, to determine their individual rates of progress over a two-year period. The results in Table 1 below relate to Algebra, in which *five* 'levels of understanding' were identified. The five levels were labelled L0 to L4.

Number of levels gained	$IQ \leqslant 89$	$90 \leqslant IQ \leqslant 99$	$100 \leqslant IQ \leqslant 109$	$IQ \geqslant 110$	Total
	n = 23	n = 22	n = 28	n = 32	n = 105
0 levels	9	14	13	6	42
1 level	10	7 (1 to L4)	14	20 (6 to L4)	51
2 levels	2	1	1 (to L4)	5 (4 to L4)	9
Regressed	2	0	0	1	3

Table 1 Number of levels gained in Algebra over the two-year period (13–15 years) in each IQ score group.

Over the two year period, no child moved up more than two out of the five levels identified (although five children moved two levels to the top level). Only about 9 per cent moved two levels. In contrast, about 40 per cent remained at the same 'level of understanding' they had attained two years earlier. So, if some progress had been made, it was not sufficient to raise the child's understanding to the next level, where he would be able to answer successfully at least two-thirds of the problem-items clustered at that level. The three per cent of children who appeared to have regressed were actually within the margin due to the standard of error of the test. The number is not, therefore, significant.

The implication of these results, which, again, were replicated in the other topics chosen for longitudinal study, is that progress in acquiring understanding of mathematical concepts is extremely slow for many children. Even if 'simplification of formulae' is the next step after 'substitution', it may require a year or more of consolidation of the latter before the child is ready to move on to the former.

Most curruiculum materials at the secondary level assume steady and significant progress from year to year. They will thus have probably forged well ahead of the stages actually reached by all but the brightest pupils, especially by ages of 14 to 16 years.

c) *The level of mathematical understanding reached by many children is relatively modest, certainly in comparison to the expectations of most curricula.*

It is impossible to go into the considerable detail of the results in each topic to illustrate this finding. A few sample results are given for five of the eleven topics.

In all cases below, the facilities (percentages of correct answers), and the percentages quoted at any given level relate only to the *14 year-old* sample. This limitation is chosen for simplification of the argument.

Number operations

Less than 10 per cent (of 14 year-olds) could consistently choose the correct expression to feed into a calculator to solve straightforward 'word problems' involving decimals. The presence of numbers less than one in the question (for multiplication) or in the answer (for division) caused the greatest problems.

For example, responses to this question:

The price of minced beef is shown as 88.2 pence for each kilogram. What is the cost of a packet containing 0.58 kg of minced beef?

88.2 + 0.58 or 0.58 ÷ 88.2 or 88.2 – 0.58 or 0.58 – 88.2 or 88.2 ÷ 0.58 or 0.58 x 88.2?

gave a facility value of only 21 per cent.

Ratio

About 55 per cent (of 14 year-olds) could, in general, apply proportionality only to problems involving a ratio in the form of 1:n or n:1, where n is a small whole number. In other words, they could only treble, double, halve, etc.[9]

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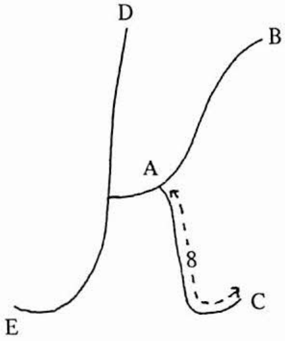
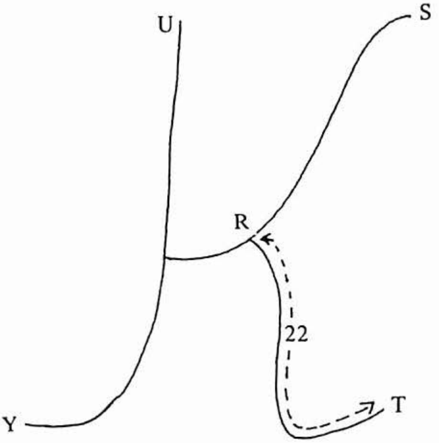
[9] Hart, K.M. The Understanding of Ratio in the Secondary School. *Mathematics in School*, Vol. 7, No. 1, January 1978.

For example, responses to this question:

These two letters are the same shape. One is larger than the other. AC is 8 units. RT is 12 units.





The curve AB is 9 units. How long is the curve RS?

gave a facility value of only 20 per cent.

Place value and decimals.

About 55 per cent (of 14 year-olds) could understand and use decimals, provided there was no need to relate the figures in the different places. Dividing by a larger number (as in the area of number operations) caused particular problems.

For example, responses to this question:

Add one tenth

2.9 → ....

gave a facility value of 51 per cent

But, responses to:

Divide by 20

16 → ....

gave a facility value of only 25 per cent.

Algebra

About 65 per cent (of 14 year-olds) were able to use letters in generalized arithmetic in cases where a number could be readily substituted to solve the problem, or where the letter could be thought of as representing an object. Some 35 per cent could also use letters where they represent specific, but undetermined unknowns, or generalized numbers. But only about 5 per cent seemed to understand the meaning of a numerical variable.[10]

For example, the responses to this question:

> Blue pencils cost 5 pence each and red pencils cost 6 pence each. I buy some blue and some red pencils, and altogether it costs me 90 pence. If b is the number of blue pencils bought, and if r is the number of red pencils bought, what can you write down about b and r?

gave a facility value of only 11 per cent.

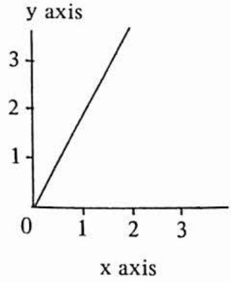
Graphs

About 40 per cent (of 14 year-olds) could *at most* plot and read information, without interpolation, from block graphs (bar charts) and scattergrams. Only 10 per cent could understand the relationship between a graph and the algebraic expression it represents, although most English children would meet this when they are 12 years old.[11]

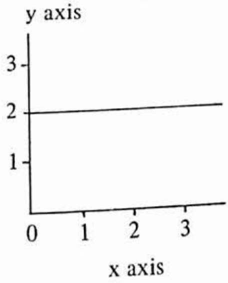
For example, responses to these questions:

> Which of the graphs below represents the line $y = 2x$?
> Put a tick by the graph you choose.

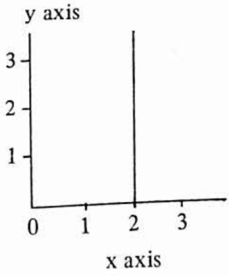
(a)



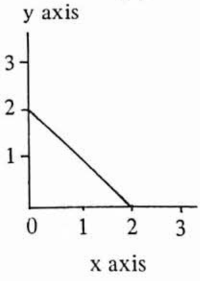
(b)



(c)



(d)



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[10] Kuchemann, D.E. Children's Understanding of Numerical Variables. *Mathematics in School*, Vol. 7, No. 4, September 1978.

[11] Kerslake, D. The Understanding of Graphs. *Mathematics in School*, Vol. 6, No. 2, March 1977.

What are the equations of the other lines?

(a) . . . .

(b) . . . .

(c) . . . .

(d) . . . .

gave facility values as follows:

(a) 18 per cent (b) 10 per cent (c) 10 per cent (d) 4 per cent.

All of the above topics and questions relate to areas of the English curriculum which would generally havebeen studied by 14 year-old children, many of them over a period of several years.

The implications of the findings are that some parts of the English mathematics curriculum appear to be quite inappropriate for the majority of children at the time when these items are introduced, for little of what was taught appears to be understood. There are two areas in which a clear mismatch shows up over a number of topic studies. One is the understanding of the nature of algebraic variables (algebra, graphs, vectors). The other is operations involving numbers less than one (decimals, fractions, ratio, number operations, graphs).

(d) *The relative independence of teaching and learning*

Although there were large variations between the mean levels attained in different schools, these differences were nearly all accounted for by the differences in the mean IQ of the school intake. Although the results have not yet been fully analysed, it would appear that they are but little affected by the considerable variations of content and method which are to be found among the different courses taught in English schools.

This conclusion is also supported by the fact that the test instruments have been tried out in other countries (Greece, Canada, and the Federal Republic of Germany) and the results reported so far do not differ greatly from the English ones. This, however, is not true of the tests of 'skills' which were used for comparison.

An explanation of the fact that the type of teaching seems not greatly to affect the learning is that a widespread use of informal methods – both successful and unsuccessful – was found. For instance, in the area of ratio, the unsuccessful 'addition' strategy[12] was used at some time by over 50 per cent of the 14 year-old sample – and in all schools. Few children, in the ratio questions, used a 'school-taught' method (for example, multiplying by a fraction, using the formula $\frac{a}{b} = \frac{c}{d}$, or 'practice'). Even the most successful pupils used some sort of informal addition-based 'building-up'. Similarly, in the area of number operations, many of the children interviewed used a mental form of 'counting-on' (complementary addition) to obtain answers to 'subtraction' problems.[13]

At the same time, some of the brightest children appeared to be able to cope with problems in topics (e.g. Vectors) they had not 'officially' been taught, provided a minimal explanation was available.

The implication of this general finding is not necessarily that the content of mathematics courses and the method by which it is taught are unimportant. It would, however, seem that variations due to different methods being used within schools in our sample were small compared with the variations due to measured IQ. Consequently, a careful control of variations due to 'inability' would have to be made before the effects of different teaching styles or course materials could be compared.

The overall conclusions of the CSMS programme were then that many of curriculum materials which are in widespread use in England and Wales are not well-matched to the level of mathematical understanding possessed by many of the pupils, and that much more attention needs to be paid to assessing the level the individual learner has attained before prescribing the next step. This is likely to maximize his learning, and, in turn, satisfy his emotional need for successful mastery of new concepts. This is just as true for the bright child as for the less able one.

Summary

It is suggested that curriculum development both in the past, and at the present time, tends too often to ignore the needs of the child as they are perceived by him in the classroom. This is true of his long-term needs, where his perceptions may be very different from those of curriculum planners, and where a greater degree of dialogue is required between all parties concerned in the mathematical education of the child. It is equally true of the child's need to be presented with short-term goals which he can see to be worthwhile, and in the attainment of which he expects to derive emotional satisfaction. The teaching method may go some way to assist, by capitalizing on the child's social, aesthetic, emotional and cognitive needs. But of overwhelming importance is the need for the child to perceive that the short-term goal is actually attainable. Research quoted suggests that secondary mathematics curricula, at any rate in England and Wales, are too often out of step with the mathematical level of the learner, and hence that the emotional satisfaction of significant learning may be denied to many of our children.

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[12] Piaget, J.; Grize, J.B.; Szeminska, A.; Bang, V. *Epistemolologie et Psychologie de la Fonction.* Paris, Presses Universitaires de France, 1968; Karplus, R.; Karplus, E.; Formisano, M; Paulsen, A.C. *Proportional Reasoning and Control of Variables.* Berkley (Calif.). Advancing Education through Science Oriented Programs, Report ID–65, 1975; Hart, *The Understanding of Ratio . . . . op. cit.*

[13] Brown, M; Kuchemann, D.E. "Is it an Add, Miss?" *Mathematics in School*, Vol. 5, No. 5, November 1976 and Vol. 6, No. 1, January 1977.

Desmond Broomes

Goals of Mathematics for Rural Development

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The Context

In 1970, some 52 per cent of the world's population were making their living from the land, or were members of farming families. It, therefore, seems reasonable to expect that, within any country, the education and training of the rural population should loom large in its plans and programmes. Further, careful analysis of population figures reveal the extent of the problem for developing countries. The world agricultural population varies considerably from region to region, and even within

Table 1: Agricultural and Non-agricultural Population in millions and Available Land per Unit of Population: 1962–1985.

	Agricultural Population		Non-agricultural Population		Arable land per unit of Agricultural population in hectares		Arable land per unit of Total population in hectares	
	1962	1985	1962	1985	1962	1985	1962	1985
Africa South of Sahara	165	250	36	107	1.10	0.89	0.90	0.62
Asia and Far East	583	880	250	591	0.45	0.31	0.31	0.19
Latin America	99	144	127	289	1.49	1.32	0.64	0.43
Near East and North Africa	88	114	47	140	0.83	0.72	0.54	0.32
Total	935	1 388	460	1 127	0.72	0.56	0.48	0.31

Adapted from: FAO, Provisional Indicative World Plan for Agricultural Development: Summary and Main Conclusions. (Malassis, Louis. *The Rural World: Education and Development.* Paris, The Unesco Press, 1976, p. 15.)

regions. For example, in North America, in 1970, one person in twenty-three was a farmer; in Europe, the proportion was one in five; in the USSR, it was one in three; in Central and South America, one in two; but in Asia and Africa, it was two in three.

In developing countries, the agricultural population is much larger than the non-agricultural population. Table 1, above, contains data which show that in certain areas the agricultural population was as high as 67 per cent of the entire population in 1962, and, by 1985, according to the Food and Agriculture Organization of the United Nations' (FAO) forecast, the agricultural population will still be about 55 per cent of the entire population.

Further, the amount of arable land per unit of population for the developing countries is expected to be reduced from 0.48 hectares in 1962 to 0.31 hectares in 1985. In some developing countries the situation is critical and serious. In Asia and the Far East, the amount of available arable land was already 0.31 hectares, and was expected to be reduced to 0.19 hectares by 1985.

Other factors have tended to emphasize the serious issues involved. In the early 1970s the FAO had warned of an impending food crisis, and, by 1974, famine was rampant in many developing countries all over the world. The so called 'Green Revolution', which started in many developing and poor countries during the 1960s, made use of high yielding varieties of plants which, in most cases, needed not only more water and more fertilizer, but also more farming expertise among persons who were usually illiterate or ill-educated, and, hence, superstitious and resistant to change. However, by the end of the 1970s it was widely agreed that the food problem was more one of distribution and of poverty than of food output. Thus at the beginning of the 1980s, *Time* magazine[1] reported that in the *United States* one farmer could seed 300 acres a day, and that the 'Plains of Plenty' produced, during 1978-79, 193 million metric tons of corn, 58 million metric tons of wheat, and 61 million metric tons of soybeans out of the total world production of 406, 403 and 95 million metric tons respectively.

A major concern in this paper is with the rural population of the world about whom it is reasonable to make certain statements, even though the available statistics on rural populations are unsatisfactory, in that the definition of 'rural' as 'non-urban' has varied from country to country and even within a country, and the collection of valid and reliable data depends to a great extent on the members of the population being able to do things expected of a literate population.

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[1] "The Plains of Plenty". *Time*, 21 January, 1980. p. 13.

First, it can be asserted that in any country the rural population is much larger than the agricultural population. Those living in the country-side include not only farmers and the families of farmers, but also merchants, clerks, factory workers, craftsmen and a whole band of unemployed country dwellers.

Second, figures from the *United Nations Demographic Yearbook* (1971), give useful, although limited, insights into size and distribution of the rural population. Of the thirty largest African countries, twelve have a rural population of over 90 per cent, eighteen of over 80 per cent, twenty-two of over 70 per cent and almost all of over 60 per cent. In the largest Asian countries, India, Indonesia and Pakistan, more than 80 per cent of the population is rural. In Latin America, which, comparatively, is highly urbanized, there are as many country dwellers as town dwellers.

Third, a threefold related finding is that the majority of people in the world's less affluent countries live in the country-side; that an important segment of the population of every nation may be defined as rural; and that the problems of the rural population of Central Africa, South East Asia and the Andes are, at first glance, very different.

Thus, in this paper, the rural world is defined to include all those developing countries where 70 to 95 per cent of all people live and work in rural areas, together with those segments of developed and affluent countries where rural (in particular, agricultural) activities are prominent.

These segments vary considerably in size, and in their overall development as measured by urban criteria. This rural world may be characterized in the developing countries by:

- high rates of demographic growth among the rural population;
- agricultural methods which are traditional, archaic and superstitious;
- the fact that most farmers and agricultural workers are ill-educated or illiterate;

and in all countries, developed and developing, by:

- increasing numbers of illiterate peasants in the rural communities;
- a lower level of schooling (and quality of schooling) among the rural community than among the urban community;
- the deterioration of the natural environment.

It is clear, therefore, that the problem of the rural world is a world-wide problem. Few nations, whether developed or developing, rich or poor, seem to be satisfied with the activities they have planned and conducted in the context of education and training for rural development.

The Nature of Rural Development

A constraint upon rural development in most developing countries is the observation, 'those who hold a pen earn more than those who wield a hoe'[2]. This single circumstance exerts immense influence on the entire community. Young people and their parents lay great store by that academic type of education which they assume will lift them out of poverty.

Thus, in most developing countries, a simple formula for education evolved. Primary education was organized to impart literacy and numeracy to the most able boys and girls. Less favoured primary pupils were kept busy doing handicrafts and gardening. Within a few years they dropped out of school. Secondary education was for the acquisition of 'school certificates'. Its major objective had little to do with the acquisition of certain skills or attitudes.. And university education was intended to enhance the earning powers of an 'educated' group. One thing was certain: the accepted philosophy of education had nothing to do with the social and economic reconstruction of the society.

Rural development, which, by our definition, takes in the development of the entire community of developing countries, could be thought of as the consequence of finding solutions to the problems of irrelevant school curricula, inappropriate community values, sterile educational philosophies and complex employment practices. The development of rural communities should attempt to fuse the services provided in such fields as health, welfare, nutrition, education, public works and agricultural extension, into a co-ordinated effort in each locality designed to stimulate initiative, self-help and mutual aid among villages.

Necessary twin conditions for the type of rural community development being described are that the value system of the community should be re-interpreted and that the education of adults and the schooling of their children should be regarded as two facets of a single operation. This derives from the realization that the qualities which are required for innovation in rural areas cannot be developed by tinkering with the curriculum of schools. The supporting evidence is abundant.

- The poor quality of the rural teacher is notorious. The defects in his education and training make it almost impossible for him to apply general scientific concepts to related phenomena in the environment. What the teacher knows outside of his experiences is what has been taught him in a textbook way.

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[2] Castle, E.G. *Education for Self-Help: New Strategies for Developing Countries.* London, Oxford University Press, 1972, p. 60.

- Agricultural education is often regarded as an appendage to 'proper' school work, or as an activity for the weaker students, or as an activity for poorer rural schools.
- The gap between the thinking of educational planners and of educational practitioners is wide. Educational planners seek to use the school curriculum as as instrument of reform. However, no school curriculum has yet emerged which combined 'subjects' with those activities that men and women engage in at their work-places and with the human relationships that are critical in the growth and development of any community.

Thus it seems fair to state that the ruralization of the curriculum would not emerge from changing the curriculum, here and there, now and then, but by evolving a method of teaching and learning modelled upon the attitudes, skills, and work-habits that are desired in the village citizens.

Redefining Schools in Rural Communities

'All societies face the task of preparing their young for full membership in the adult community and all cultures include education as the principal means of perpetuating themselves.'[3]

Schools have become the chief agent of educating the young. They exist in varying degrees of 'schoolness'. They range from the unconscious observations that children make of elders in the family to the more organized and formalized school situations.

In preliterate societies, children learned by doing.

'The homestead was the school. In the homes and on the farms, the children were taught the skills which would enable them to play their full part as adult members of their society. They learned by direct observation and imitation. By this method, the boys learned how to distinguish useful grasses and dangerous weeds, how to stalk wild game, and how to stock sheep and goats; and the girls were similarly taught the skills related to their sex and age, such as the preparation of meals and looking after babies. All children were taught tribal history by oral tradition and were also helped to acquire the sacred cultural mores and attitudes as well as the modes of behaviour which were valued by their society.'[4]

[3] Kajubi, W. Senteza "Is the school an Obsolete Institution?" In *Educating the Young People of the World*, edited by Alice Miel and Louise Berman, Washington, D.C.: Association for for Supervision and Curriculum Development, National Education Association, 1970, p. 9.

[4] *Ibid.* p.9.

An education as described by Kajubi had obvious advantages. First and foremost, it related directly to the needs and problems of the society. Second, it involved more or less all members of the society, every child as a learner, every adult as a teacher, and every institution as part of the education process. Third, it ensured that persons developed as emotionally stable and economically productive members of the community.

This education, however, had some severe limitations. All children were given, more or less, the same kind of instruction according to their sex and age. The education depended heavily on custom and tradition. So it was oriented towards the past. In the teaching-learning process, memorization and recall of information were emphasized, while innovation and creativity were often ignored.

As societies became more complex, the processes of socialization and enculturation became more intricate and specialized. Thus evolved the formal school with its responsibilities for ensuring the orderly transmission of culture from generation to generation, and for educating the young. And herein lies a major problem. 'Whereas, from one point of view education is a specialized study, with its own techniques, and is the business of experts, from another point of view education is everyone's business and involves every thing that living itself involves. The process of intellectual, emotional, and social growth is susceptible of expert investigation, and affords a valid group of appropriate disciplines. At the same time, none of us can escape active participation in the business of education, our own and other people's, since the influences of persons upon one another must result in education of some kind, good, bad or indifferent.'[5] In developing countries, the way the schools evolved created a second (and a larger) problem. The African continent (or the Caribbean crescent) can serve to illustrate.

In most African countries, the first schools were established by missionary societies. They had two main objectives: first, to save the souls of the natives, and second to train clerks, artisans and other workers for the economic activity stimulated by the colonials. The values and skills taught in these schools were frequently at variance with practices at home and in the community. The values of the teachers also differed from the values of the parents of the learners.

Subsequent political independence and a strong national spirit stimulated the development of curriculum materials based on the local environment, the incorporation into the school curriculum of indigenous music, art and folklore, and a bridging of the gap between home and school. Thus in most developing countries education became a major political concern and expenditure on education has been maintained at about 25 to 33 per cent of annual budgets.

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[5] Jeffreys, M.C.W. *Glaucon.* London, Sir Isaac Pitman & Sons, 1950, p. 3.

Yet there are serious problems in developing countries: only 50 per cent of children of primary school age are receiving more than a few years of schooling; primary education is generally academic and literary, aiming at preparing children for secondary school (and secondary education for the university); only 2 to 4 per cent of children of secondary school age can be accommodated in secondary schools; schools, by virtue of their limited accommodation and resources, nuture among children a highly competitive and selfish spirit – as against the self-help and national co-operative movements being fostered by the community; teachers are inadequately educated and inadequately trained, and many are untrained; teaching methods are formal, emphasizing drill and the memorization of information; and the curriculum removes the children from their social and cultural milieu, emphasizing book learning and examinations.

Schools as we know them, especially schools in developing countries, are not organized, equipped and run to combat these problems.

First, school facilities cannot be expanded in the same way as the school plant has been built and used in the past. It would require educational budgets absorbing 50 to 70 per cent of national budgets to provide all children of primary school age with comparable facilities.

Second, those in school take a long time to become adult men and women with employable skills. Their impact upon economic and social development will not be felt by the community until at least two decades later.

Third, the school plant and its services are heavily under utilized. Schools open at 8 o'clock in the morning and close at 4 o'clock in the afternoon. Furthermore, they open for only nine months of the year. Schools are not concerned with the education of adults.

The concept of a school must therefore change. It should broaden into that of an institution integrated with the community as a whole. It must be concerned with the education of parents as well as of children. It must be concerned with the education of the men and women who produce our food, our clothes and our shelter. It must operate so that other institutions – economic, social, and political – can contribute to the education of the young. Education must become a flexible process – no longer tied to time (pupils 5–16 years old), to place (buildings conventionally called schools) and to a methodology (pupils doing assignments set by teachers).

Re-analysis of the data obtained from rural communities leads to five propositions which would allow rural growth and development to be interpreted in economic, social and cultural terms. The propositions may be stated as:

- education should be conceived as a continuing process from infancy through adulthood;
- the needs of the users of the educational institutions must be the starting point of any analysis;
- the needs and values of the community should inform the methods and content of education;
- all institutions (economic, social, educational) should be organized to play crucial roles in the education of every citizen;
- education should be conducted using a variety of methods and materials.

The first two, and the last of these propositions were identified by Coombs.[6] They allowed him to group the various types of learning within any community into three categories:

- *Informal education.* A relatively unorganized and unsystematic process by which every individual acquires attitudes, values, skills and knowledge from the daily experiences and educative influences in the environment.

- *Formal education.* A hierarchically structured and chronologically graded process which runs from primary schools to secondary schools to universities and includes academic, technical and professional studies.

- *Non-formal education.* Any organized educational activity outside the established formal system that is intended to serve specified learners and learning objectives.

Coombs' conceptualization of education in terms of informal, formal and non-formal components, overlapping and interacting, seems a very useful way to think about the educational programmes of any community. It also provides a useful frame of reference for assessing and planning effective programmes.

Three trends which characterize the role of education within the process of rural development, are, according to Coombs[7], self-evident.

✓First, rural development means more than simply increasing agricultural production. Rural development means *rural transformation*. It requires increased productivity, equitable distribution of income, increased employment, land reform, better health, nutrition and housing, expanded educational opportunities, strengthening of political institutions and the promotion of social justice.

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[6] Coombs, Philip H. "Non-formal education in rural development", in *Educational Development International*, Vol. 2, No. 4, 1974.

[7] *Ibid.*

Second, education cannot of itself ensure rural development. 'Education must be seen as only one input among many (inputs for rural development), its impact depends not only on its own quality and relevance, but on its interaction with complementary inputs.'[8]

Finally, rural development must be viewed in the context of national development, and of the interactions between rural and urban economies. It must be defined as an entity with its own integrity and not as an accidental spin-off from urban/industrial growth.

Coombs concluded: 'It seems clear from the evidence gathered in this study that the educational deprivation of rural children and adolescents in many parts of the developing world is more serious than is generally assumed. This sombre picture is relieved by evidence of promising new approaches, but nothing short of a truly massive and innovative effort over the next two decades can hope to correct the situation.'[9]

Developing countries and rural communities have, therefore, to re-examine the curriculum (its objectives and its content). The major problem is to *ruralize* the curriculum. This does not mean the inclusion of agriculture as another subject on the programme of schools. Ruralizing the curriculum means inculcating appropriate social attitudes for living and working together in rural communities. Ruralizing the curriculum must produce good farmers, but it must also produce persons who would co-operatively become economic communities as well as social and educational communities.

Goals of Mathematics for Rural Developments

The issues identified and discussed in the sections above have served to point out the psychological, social and political factors that should interact among themselves to generate a set of goals of mathematics for rural development. The main components are the *individual* (and other individuals), the *institutions* (economic, social, political) and the *community* (as a society).

By focusing first on the needs of the individual as he or she relates to other individuals, to the institutions and to the community, and then on the needs of the community as a society in a similar way, it is possible to uncover goals for mathematics education in a rural environment. The figure below illustrates the various strands of thought.

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[8] Ibid. p. 167.

[9] *Ibid*, p. 168.

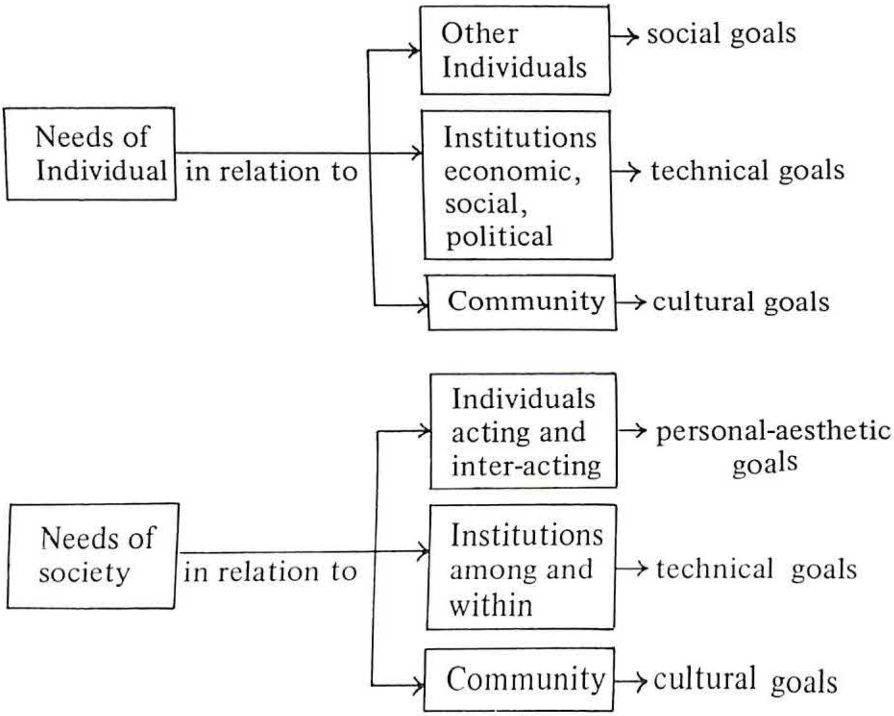

Figure 1: Generating Goals for Mathematics education

The different types of goals that have emerged may be summarized under four headings:

1. Social Goals

These are needed for:
- working with others;
- thinking with others;
- feeling with others;
- living with and among others.

2. Technical Goals

These are intended to provide:
- mathematics for understanding and studying other subjects;
- mathematics for the further study of mathematics;
- mathematics as method of investigation;
- mathematics for the working roles we have to play as housewife, farmer, lawyer, etc.

3. Cultural Goals

These are intended to provide insights into:
- mathematics as a symbolic system;[10]
- mathematics as expression of man's intellect and of man's historical development;
- mathematics as man's disentangling of Nature;
- mathematics as a way of thinking and of communicating thought.

4. Personal-Aesthetic Goals

These, broadly, are of two types:
- the personal types, which stimulate curiousity and intellectual excitement, and those which permit personal expression;
- the aesthetic types, which aim to convey an appreciation of mathematics as a deductive system and of mathematics as concerned with patterns and shapes and order.

These goals of mathematics for rural development have been placed under five broad headings:

1. The preparation of citizens as users of mathematics (Social Goal);
2. The contribution of mathematics to cultural synthesis (Cultural Goal);
3. Mathematics as an educative force in every person's life (Personal Goal);
4. Training of mathematicians and other mathematical scientists (Technical Goal);
5. Mathematics as a science with its own unique characteristics and beauty (Aesthetic Goal).

Nothing has been said to suggest, even vaguely, that these five goals are mutually exclusive. Indeed there is much overlap among the activities involved in attaining each goal. That said, the goals can be more fully described.

Goal One: The preparation of citizens as users of mathematics

It is reasonable to assert that any mathematics programme should develop among the learners some basic skills of numeracy, some understanding of patterns and shape, some idea of the applications to domestic, social and day-to-day living and, to some extent, an ability to fit mathematical models to practical situations.

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[10] Phenix, Philip. *Realms of Meaning*. New York, McGraw-Hill Book Co., 1964.

However, some major problems confront school systems as they attempt to implement this goal. How should mathematics materials be prepared if they are to cater for persons who possess a wide range of mathematical abilities, interests and needs? How should classes in mathematics be organized for these persons? How should teachers in these classes teach mathematics to these persons?

The available evidence shows that the way mathematics is taught, and the materials used for teaching it, have tended to remove mathematics from the conscious minds of certain persons on whom the community depends for its welfare and well-being – the farmer, the agricultural worker and the housewife. When, for example, mathematics is taught away from the context in which the subject occurs in real life, the learners may acquire the skills needed for doing computations and simple applications, but they will seldom be able to apply their mathematics when they think about the everyday problems they meet as citizens. Curriculum development in mathematics should therefore attempt to define that body of mathematical skills, facts, concepts and principles which are needed for everyday life and to devise a suitable pedagogy for transmitting, in a useful way, this body of knowledge to every member of the community.

Briefly, the body of mathematical knowledge needed for everyday life would include much of what is now called arithmetic, algebra, geometry, statistics and logic. But the way any body of knowledge is transmitted is an integral part of the knowledge. So, even if the content used to give concrete embodiment of the concepts studied (or to motivate the study of the concepts) is not part of the body of mathematical knowledge as such, its transmission will serve to forge linkages between mathematics and its contextual use. Great emphasis should be placed on the mastery of skills and on simple applications in algorithmic form. Nevertheless, the way the mathematical skills are acquired can influence a person's way of thinking about the social, economic and political issues that swirl about him at all stages of day-to-day living.

Goal Two: The contribution of mathematics to the cultural development of society.

In any society, mathematical activity should be recognized, nurtured, and preserved as an integral part of the culture, and it should be developed as a major dialectical force in creating cultural syntheses.

The people of every society are confronted with the problem of inducting the immature members into the ways of the group. In primitive societies, the individual acquires the ways of the group from association with adults in their daily activities. The boy learns to hunt, to till the soil or to shape the tribal tools. He does this by sharing, in socially useful ways, in these activities with his elders. In literate societies, instruction in the ways of the group is done through a sequence of potential experiences organized by specialized institutions called schools. The set of experiences is usually described as the curriculum of schools. The curriculum then is a reflection of what the people think, feel, believe and do. To understand the curriculum, it is necessary to understand at least what is meant by culture.

According to Smith, Stanley and Shores[11]. a culture is the set of ideas, ideals, beliefs, skills, tools, aesthetic objects, methods of thinking, customs and institutions of a given society at any given time. The way the members of the society make a living, the games they play, the stories they tell, the heroes they worship, the music they play, the way they care for their children, their family organization, their modes of transportation and communication – all of these comprise the culture of a people.

Human beings are born into a culture; and it is the elements of the culture, acting on and interacting with their receptive nervous systems, which determine the habits, beliefs, thoughts and deeds that make human beings what they are. In any society, most persons act to some extent as transmitters, mediators and valuators of the mathematical sub-culture. They carry and sustain the mathematical traditions in the culture, in many minor ways – by, for example, doing simple computations required for shopping, borrowing money or using a recipe. This conception of mathematics traces back to our Egyptian heritage, where mathematics was a body of rules for handling quantitative problems arising in the workaday world. However, the principal mathematical elements in the culture will be possessed by professional mathematicians and by those who work in certain professions that employ a great deal of mathematics. This, in a sense, could be described as our Greek heritage.

The direction that mathematics takes in a culture is guided by the religious, philosophical, agricultural, industrial and also the mathematical components of the culture. To most laymen – as against mathematicians and mathematical scientists – mathematics is thought of as a fixed body of truth, to be learned convergently, and epitomized in such statements as

'1 and 1 are 2', and
'-1 times -1 is $+1$'.

But mathematics grows under the influence of a complexity of evolutionary and diffusionary forces. 'Given a suitable juxtaposition of ideas, either in the mind of an individual or in the minds of a group, certain syntheses take place and new concepts come forth. Mathematics does not grow because a Newton, a Riemann, or a Gauss happened to be born at a certain time; great mathematicians appeared because the cultural conditions – and this includes the mathematical materials – were conducive to developing them.'[12]

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[11] Smith, B.O., Stanley, W.O., Shores, J.H. *Fundamentals of Curriculum Development.* New York: World Book Company, 1957.

Goal Three: Mathematics as an educative force in every person's life

Mathematics should be a major component of every person's education. But this mathematics requires a new definition and its nature needs to be reinterpreted. What seems critical in the reinterpretation is to include in the content of mathematics the end product, as well as the whole domain of activities in which mathematical ideas and procedures 'germinate, sprout and take root, and in the end produce the visible upper branching, leafing and flowering which all we here so value and which wither when uprooted.'[13]

The 1960s and 1970s were remarkable for the exciting directions taken by science and technology, and for their impact on society. In particular, the 1970s were a decade of scientific revolution, from which emerged new ways of expressing the aspirations of mankind, new ideas about man's relationship to man and also man's relationship to his environment, new tools for controlling and exploiting nature and new areas of investigation and research. In all these activities, mathematics has played a central role. Thus d'Ambrosio [14] was moved to state 'The main objective of mathematical education . . . is to develop the ability of individuals to identify mathematics in intellectual experience, and distinguish mathematical reasoning and method in all situations where they are either present or can be potentially inserted.'

Today's world is explained and defined in terms of science, mathematics and technology. Every person needs to understand the world in which he lives and to contribute to its growth and development. A person needs to use the power and pattern of mathematics which are accessible to him or her through certain abilities (ability to use symbols, to reorganize patterns, to organize data, to draw conclusions) and to understand the role of the mathematician in the society.

[12] Wilder, Raymond L. *Introduction to the Foundations of Mathematics.* Second edition. New York: John Wiley and Sons, Inc., 1965. p. 293.

[13] Hawkins, David "Nature, man and mathematics". In: *Development in Mathematical Education: Proceedings of the Second International Congress on Mathematical Education,* edited by A.G.Hawson. Cambridge, Cambridge University Press, 1973. p. 43.

[14] d'Ambrosio, U. "Overall Goals and Objectives for abstracted Mathematics Teaching (Why do we teach Mathematics?)" In *Proceedings of the Third International Congress on Mathematical Education*, edited by Hermann Athen and Heinz Kunle, Karlsruhe, Organising Committee of 3rd ICME, 1977. p. 225.

Some of the implications of identifying these activities (and logically clustering them to form a goal of mathematics) touch on the mathematics programme in schools as they are currently organized (primary, secondary and further). Thus, it is necessary to focus on the content, the teaching methods, the actual teaching, the materials used and the relationship between the school mathematics programme and the activities of the community.

The point is that it is no longer enough to think of educational development (as was the wont of educational reformers during the 1960s and 1970s as primarily a matter of improving the interactions of teacher and learner, while regarding content as of secondary importance. Such reforms are far from adequate. Not only do we need to devise methods which would allow the emergence of creative behaviours among teachers and pupils in the mathematics classroom, but, most important, the methods should allow the mathematics of the classroom to penetrate other subjects of the school programme *and* the external world. Moreover, the external world should impinge on the mathematics to be studied in schools (primary, secondary and further). This last condition is necessary to ensure that the mathematics is taught in useful ways, using useful materials, to persons with a wide range of abilities and interests and needs; and that mathematics be organized and taught as 'a variable that has loadings on a convergence-divergence factor, on a syntax-substance factor, and on an objectivity-subjectivity factor.'[15]

Success in achieving this goal depends greatly upon the way teachers behave as teachers. The demands on them are severe. They must have a rounded, humane, and comprehensive view of mathematics. Their training is therefore critical. It should convey knowledge of the cultural role of mathematics, knowledge of the critical contribution of mathematics to technology and science, experience of teaching mathematics to those who do not intend to become mathematicians or mathematical scientists, and ways of teaching persons from different sub-cultural orientations. These components of the training programme seem to constitute necessary conditions to ensure that the teacher presents mathematics usefully to his pupils so that the mathematical mode of thought is revealed as one of the most elegant ways of expressing their ideas, of organizing their experiences and of finding meaning in their activities.

Mathematics as an educative force must therefore be experienced by all members of society – not merely as a counting experience to find out how much less to pay for an article which is on sale at 25 per cent discount, or to calculate income tax refund for a given year. It must give the learner some taste of the power and beauty and elegance of mathematical argument.

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[15] Steele, J.M., Walker, Herbert T. and House, Ernest R. "Subject areas and cognitive press". *Journal of Educational Psychology*, Vol. 66, 363–6, 1974.

The teaching of mathematics must also help the learner through experience to appreciate the contributions that have been made to this mode of thought through the use of symbols, tables, diagrams – the 'language' of mathematics. Through its language, mathematics has achieved remarkable precision and clarity in describing and communicating ideas. It possesses, also, tremendous flexibility in the management of ideas. This latter characteristic makes it possible for us to be creatively constructive as we think about our experiences and ideas.

For example, the concept of 'a limit' is a basic one in mathematical thinking. It is a fruitful way of thinking about certain ideas and of describing some problems. Mathematicians have therefore always felt the need for a logically sound and rigorous definition of 'a limit' in order to communicate its meaning precisely. The set of symbols which is usually used to show a limit is:

$$\lim_{h \quad a} f(h) = b,$$

The essence of this statement is that f(h) can be made to take on values as close to b as may be desired, by giving h values which are close to a. All this can be precisely stated as follows:

$$\forall \epsilon > o, \quad \exists \delta > o, \quad \forall h \text{ in the domain of } f,$$
$$\{o < | h - a | < \delta\} \Longrightarrow | f(h) - b | < \epsilon.$$

Mathematics is not merely the subject we study to become mathematicians, engineers, carpenters and sales-clerks. Mathematics is also about the subject we should study in order to participate in the thinking that pervades all society – in a good argument, in organizing our ideas, in coping with abstractions. But, to achieve this goal, the subject must be organized and taught very differently from the way in which it is usually taught.

Goal Four: Training of mathematicians and other mathematical scientists

A major goal of mathematics education is to identify persons with mathematical and scientific aptitudes, and to encourage their development. The interplay between mathematics and the sciences (physical and social) is an essential feature of mathematics, by virtue of which mathematics serves and is served by the sciences.

The subject matter of a science is a collection of sense data which at first sight seem chaotic. In search of order, the scientist seeks to find some pattern which may fit the data, more or less. This allows him to answer appropriate questions about the data and the phenomenon associated with the data. The mechanisms which are used to find these patterns and to express a pattern in a useful way, are almost always mathematical. In particular, the symbolic formulations of mathematics make unique contributions to:

- delineating a phenomenon in its generality;
- correlating data from a variety of phenomena; and, thereby, permitting new meaning to be given to certain phenomena.

Such applications of mathematics have given rise to new careers, such as those associated with operations research, information theory and statistics; and careers in which mathematics is used not only to think about, and to solve more efficiently, the problems of agriculture, economics, education, psychology and medicine, but also to gain entirely new insights into these subjects and to create new syntheses. Further, in stressing the interplay between the needs of the community and the needs and interests of individuals, it is not intended to give mathematics a narrowly utilitarian purpose, or to make mathematics a simple counting exercise – the popular conception of what is 'useful mathematics'. The interplay that is chosen for study should ensure that the learners experience high motivation, that teachers use a pedagogy that is consonant with the dominant cultural values of learner and teacher, that the content relates to the interests of the learner, and the needs of the society, and that a psychological and philosophical closeness is encouraged between schools and society. This interaction between mathematics and other subjects and between mathematics and the external world can be effectively sustained through project work in depth, or by the study of real-world practical problems. Such project work provides the learners with insight into 'model building', insight into the bridge between abstract world of mathematics and the real world, and confidence in mathematics as something they can use as members of society.

Lighthill[16] has argued that, in each society, the development of mathematics curricula needs to take into account the interaction between mathematics and that society, and to bring out the fact that the major use of mathematics is not only in the analysis of problems, but also for synthesis, as in engineering design or in the statistical design of experiments. Thus Lighthill identifies:

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[16] Lighthill, James. The interaction between mathematics and society. In *Proceedings of the Third International Congress on Mathematical Education*, edicted by Hermann Athen and Heinz Kunle. Karlsruhe, Organising Committee of 3rd ICME. 1977. p. 31.

- a mathematics of the environment, in which mathematicians have formulated and used mathematical models of environmental processes and developed effective computable algorithms to implement those models because 'we care about the air we breathe . . . .';
- a mathematics of efficiency, in which mathematicians have designed ways of improving the utilization of resources and of optimizing tactics because 'we care about jobs . . . ';
- a mathematics of planning in which mathematicians have defined the dimensions of a system, because 'we depend also upon trade'.

Goal Five: Mathematics as a science with its own unique characteristics and beauty

Mathematics, as an invention of man's mind, seems to resemble closely the workings of nature. As Langer[17] put it: 'The relationship of one variable to another, which is the essence of the definition of the function, is discernible in all manifestations of nature, and no less in the workings of such man-made disciplines as economics, education, and politics.' . . . . . 'It is because of the ubiquitous prevalence of functional relationships that mathematics insinuates itself into all fields of human thought.'

Over and beyond these important contributions, the study of mathematics contains exciting vistas which seem to reside in the subject itself. In other words, the beauty, the internal structure and the pattern of mathematics contain unique and significant features that make mathematics a study worthwhile *in itself.* These features may be delineated in many ways, two of which will be sufficient for the scope of this paper.

A unique contribution of mathematical thinking is mathematical proof. Mathematical proof goes much further than merely corroborating practical experiences; mathematical proof raises completely new questions and creates completely new syntheses.

The Pythagorean discovery of incommensurables is a classic example of this. No amount of piling up of practical examples, no arguement of the practical man was sufficient to have led to the discovery of incommensurables. The discovery that $\sqrt{2}$ could not be expressed as a ratio of two whole numbers served to convince Greek thinkers, as far back as 2,500 years ago, that mathematical reasoning should loom large in a person's intellectual growth and development.

Again, when studies in any field or subject area are pressed beyond the primitive and rudimentary stage, mathematics is brought into the field as a tool. The studies are thereby lifted from the empirical level, where the major preoccupation is with observations (that is, the collection of the sense data) to an intellectual level, where the search for underlying patterns of order is the primary concern. Then, after gleaning the subject matter of mathematics from the subject matter of the field, the mechanisms of mathematics can be brought to bear. The outcomes, not infrequently, can be entirely unexpected and delightfully surprising. Thus, for example, the mathematical analysis of the speed at a particular instant of an accelerating body turns out to be equal in value to the gradient of the tangent drawn to a smooth curve. Conversely, the distance travelled by an accelerating body is found to be equal in value to the area under a smooth curve. Such results could scarcely be foreseen. This element of 'surprise' in mathematics is one of its unique as well as one of its most attractive features. The possibility of experiencing this refreshing characteristic is part of the human birthright. It should be passed on.

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[17] Langer, Rudolph E. "Functions". In *Insights Into Modern Mathematics.* Washington (D.C.), The National Council of Teachers of Mathematics, 1957. p. 241.

Conclusion

The goals of mathematics for rural development have been motivated through two major activities. First, by careful analysis of the rural problem, it was possible to describe the failure of well-meaning rural development projects in terms of cultural alienation, of conflicting reward systems among the different classes of the society, and even of a symbiotic relationship between the poor and the rich. Second, by defining schools in broader and more realistic terms for rural communities, it became morally right to give schools an explicit role in (a) creating cultural syntheses within the community and (b) equipping all persons in the community – young and old – with useful skills for producing food, clothing and shelter, and allowing them to learn these skills within the economic, social and political institutions of the nation.

Between, and within, these two major activities, the nature of mathematics emerged, protean-like, as a tool for other subjects and activities, as a method of investigation, as a language for communication, as a record of man's strivings, as a model of co-operative action, as an object and activity of its own beauty, as a foundation for further study for some persons, as skills and knowledge for everyday living, as an opportunity for experiencing intellectual excitement; in short, as a way of thinking, feeling, and doing.

Goals of mathematics for rural development are intended to reflect the nature of the rural problem, and the nature of the schools of the 1980s.

Ruth Rees

School Mathematics: Links with Commerce and Industry

Introduction

There have been changes in British schools during the last two decades, the most dramatic of which has been the reorganization of the secondary schools into a comprehensive system. There have also been changes in the content of the school mathematics curriculum.

These changes have been accompanied by a growing public awareness of the need for 'links' between the mathematics taught at school and that required for employment and general living.

This paper will discuss some of the ways in which these links are being established. An understanding of the educational systems in which such links occur will enable the reader to better appreciate their nature.

The systems, and the curricula of the schools described, are those of England and Wales: Wales has a measure of independence, but is closely linked with the English system. The systems in Scotland and Northern Ireland, whilst similar in certain aspects, are separately administered, and the curricula of the schools differ in detail from those of England and Wales.

Educational Systems

The School System

Most children start school when they become 5. They continue as primary school pupils until they are aged 11 or 12. They then transfer to a secondary school. Local Education Authorities (LEAs) build and administer the schools, and they employ the teachers. In law, LEAs are responsible for the school curriculum, but, in practice they delegate this responsibility almost entirely to each school. Thus, teachers in England and Wales 'enjoy' a freedom not shared by many of their colleagues in other countries. The independent schools, which are not under the control of the LEAs, cater for about 5 per cent of all school children.

The role of the Central Government is assumed by the Department of Education and Science (DES). It is not easy to define. DES is responsible for decisions on overall policy, and for co-ordinating the work of the LEAs, but it has no direct control over school curricula. Any major changes in policy have to be approved by the Secretaries of State for England and Wales.

A central role in the development of the curriculum is played by the Schools Council. This is an independent body, financed partly by the Central Government and partly by the LEAs. The Schools Council initiates research, and funds development projects. Its large number of committees provide a forum for discussion between representatives of those associations of teachers with an interest in the school curriculum.

There are two major systems of external examination to cater for the different ranges of ability. The first is the General Certificate of Education (GCE). This operates at Ordinary Level ('O') for pupils aged 16, and at Advanced Level ('A') for pupils aged 18. The GCE 'O' level caters for the most able 25 per cent of the population. There are eight independent GCE Examination Boards for England and Wales, with committees on which teachers are represented. They provide a range of syllabuses from which teachers select. These examination boards have links with the universities, and their examinations are used as criteria for entry into higher education (i.e. into universities and polytechnics).

The second system, the Certificate of Secondary Education (CSE), was intended to provide examinations for approximately the next 40 per cent of the ability range. In practice, since all pupils have to stay at school until the age of 16, a greater percentage take CSE. There are fourteen CSE examination boards. These boards encourage teachers to design their own mathematics syllabuses, and to set their own examinations, if they so choose. An external moderating procedure ensures comparability of standards.

It is intended to reorganize the present systems of examinations so that, by 1986, there will be one common examination system for 16 year olds. It is expected that alternative papers will be offered to cover the wide range of ability examined.

The Continuing Education System for 16 to 19 year olds

At the age of 16, students have a wide variety of routes if they wish to continue their education. Some stay at secondary school to continue their studies in the 'sixth form'. Others continue at a separate 'sixth form college'. Yet others go on to full-time education in technical (or further education) colleges. These traditionally have provided vocational education. But they are now increasingly providing as well the kind of courses available in schools. The majority of 16 year olds, of course, leave school for employment. But this may require part-time education at colleges of further education.

When pupils enter college they are called students. Sixth form school pupils or college students with appropriate qualifications may be accepted for a university course. The term 'continuing education' is now used to include both 'higher education' (in universities and polytechnics) and 'further education', described in the next section.

The Further Education System (Vocational)

This system provides one of the strongest links between education, commerce and industry. Many pupils, on leaving school, go immediately into employment, but continue their education on a part-time basis. This usually means that they are 'released' from their employment to study for one day each week at a college of further education. The education they get at such a college is vocationally orientated, but it does include some 'general' studies. The vocationally orientated element harmonizes with the training given by the employers. Many students who were unsuccessful at school feel that the part-time day-release system of education provides them with a new opportunity to study with a specific training goal in view.

One of the largest and oldest examining bodies is the City and Guilds of London Institute (CGLI)[1]. It is concerned with testing and certifying competence in technical and vocational jobs of all sorts. CGLI courses have been developed for some 300 different vocational subjects for colleges and schools to use not only in the United Kingdom but also overseas.

CGLI also provides 'Foundation Courses'. These can be taken by pupils in full-time education at secondary schools for whom GCE 'O' level or CSE courses are not entirely appropriate. Such courses are available in Agricultural Industries, Art and Design, Computing Studies, Engineering, Nursing and Allied Professions, Public and Community Services and many others. Other courses leading to the award of certificates are also available at the early employment stage.

One characteristic of the CGLI courses is that such mathematics as may be involved is not taught or examined as a separate subject, but as an integral component of the course. The aim is to ensure that the mathematics is used in the context of its need.

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[1] *City and Guilds of London Institute, What it is and what it does.* Published by City and Guilds of London Institute, 76 Portland Place, London W1N 4AA.

One exception to this rule is the new course offered in numeracy. This course has been designed to improve the numerical skills needed for most industrial and commercial operations. It may be taken prior to, or in parallel with, a vocational course. It could thus form a new and useful link between school and employment.

To embark upon a CGLI course, it is not normally necessary for someone who leaves school at 16 years of age to have obtained a GCE 'O' level pass or a CSE grade. Traditionally, an employee who is accepted to train as a skilled craftsman will leave school with a CSE grade 4 or less. Now, however, with rising unemployment, employers can be more selective.

The Royal Society of Arts (RSA)[2] is another long-established examining body. RSA conducts examinations principally in business studies, including secretarial skills, and languages. The mathematical ingredients, mainly arithmetical, are examined separately, but in the context of business studies.

Two new examining bodies have emerged during the last decade to develop appropriate courses and qualifications for technician and business studies. These are the Technician Education Council (TEC)[3] and the Business Education Council (BEC)[4].

TEC courses provide a qualification for students employed in technical fields who aspire to supervisory duties and to similar posts of responsibility. BEC courses likewise provide for students employed in commerce and business. Pupils leaving school who wish to enter a technician course are required to have a minimum qualification of CSE grade 3.

TEC and BEC, being both new, are, for the time being provided with administrative services by CGLI. This arrangement provides a useful link between the various categories of vocational courses.

Figure 1 below attempts to summarize in pictorial form what has, so far, been said about the school and further education systems. It may also assist the reader to appreciate the discussion of the mathematics curriculum in schools which now follows.

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[2] Royal Society of Arts, 18 Adam St., London WC2N 6AJ.

[3] Technician Education Council, 76 Portland Place, London W1N 4AA.

[4] Business Education Council, 168-173, High Holborn, London WC1V 7AG.

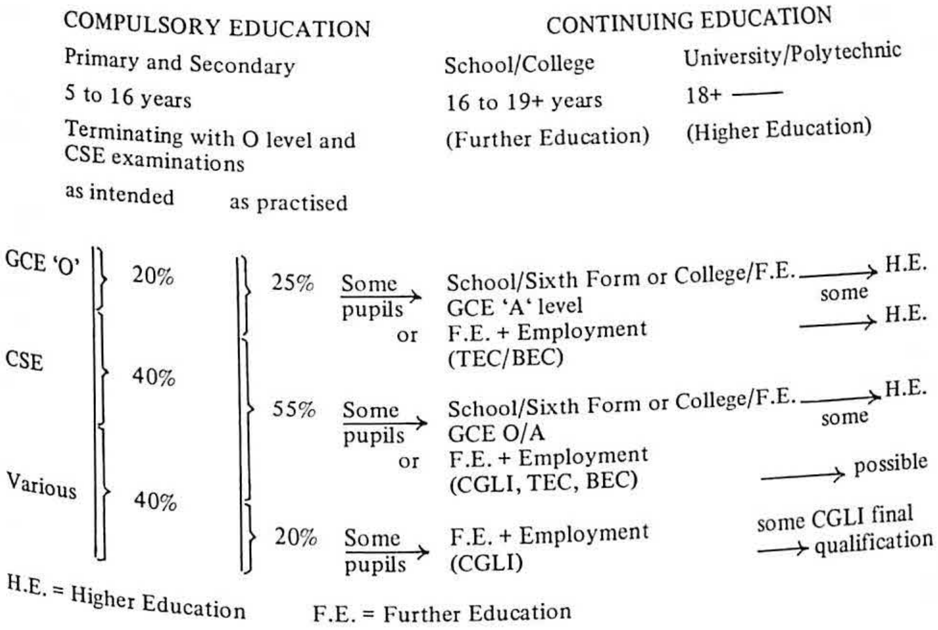

Figure 1 (Simplified) showing routes between Educational Systems in England

School Mathematics

Curriculum Development

Mathematics is an important subject of the curriculum and must be considered in relation to the school system. Teachers have freedom to choose content, methods and teaching styles. A great variety of mathematics teaching therefore exists, and the examining boards cater for this by offering different syllabuses from one another, and also by offering themselves a number of alternative syllabuses.

Attempts have been made to effect major changes in mathematics curricula during the last two decades both at primary and secondary school levels. At primary level, several curriculum projects have been launched, including the Nuffield Scheme. This was designed to improve children's understanding of mathematics by encouraging their participation in mathematical activities.

There have also been several projects at the secondary level. Possibly the most widely adopted 'modern' syllabus is that of the School Mathematics Project (SMP). This sought to integrate arithmetic, algebra and geometry, and to present mathematics as one subject. This project was originally an enterprise of the private sector of education. Its adoption by state schools may be a sign of the need of teachers for a consistent series of text books based on one syllabus.

It must be stressed that the role of teachers in curriculum development has been crucial. Many teachers have been creatively involved in the changes introduced.

The greatest factor necessitating the need for a 'fresh look' at mathematics in schools has undoubtedly been the reorganization of secondary schools on comprehensive lines. This has exposed specialist teachers, many of them for the first time, to the whole ability range of pupils. Previously accepted content and ways of teaching the more able pupils in grammar schools were not necessarily applicable to the average and less mathematically able pupils. To attempt to meet the wider range of needs, different methods of teaching have been developed. There has, for example, been a move away from the text book towards the use of work cards for individualized learning. SMP, for example, provides both work cards and a series of text books for the 'O' level and CSE examinations. The newest project to emerge commercially is the Kent Mathematics Project. This project seeks to provide an individualized course in mathematics for each child within a flexible scheme, which also provides teachers with a means of assessment and diagnosis. It caters for the age-range 9 to 16.

The Current Position in Mathematics

The current state of mathematics in secondary schools is described in a recently published report[5] of Her Majesty's Inspectors (HMIs) of Schools. This was the account of an exhaustive inquiry into what actually happens in the curriculum in secondary schools. Mathematics was one of the main components upon which interest was focused. The method of assessment was based chiefly on normal inspection methods of observation in the classroom and about the school, discussion with teachers and pupils and scrutiny of recorded work.

Some of the observations on mathematics may be summarized as follows:

(i) Mathematics is one of two school subjects (English being the other) which have a universally recognized place in the 'common core' for all pupils. It is a regularly timetabled subject for all pupils during the first three years of secondary education, and, except for a very small number of pupils, continues to be a compulsory subject in the fourth and fifth years, when pupils are provided with varied patterns of courses.

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[5] *Aspects of Secondary Education in England.* A survey by HM Inspectors of Schools, Department of Education & Science. HMSO.

(ii) Some 80 per cent of all pupils were following courses leading to 'O' level or CSE examinations. (Although CSE was not designed to cater for pupils below about the 60th percentile of ability.)

(iii) The design of courses which can cater for the whole range of ability still presents problems, particularly at both ends of the range of ability.

(iv) Almost all schools give adequate time to teaching and to practising the basic skills, but a concentration on the skills in isolation from application or any meaningful context does not always lead to mastery of the skills or confidence in using them. In the majority of classes the teaching does not aspire to doing more than prepare the pupils to answer examination questions. Mathematics is generally too little related to the outside world or to other subjects in the school curriculum.

(v) The effect of modern courses on competence in arithmetic has aroused public interest. In an analysis of the extent to which pupils following traditional, modern or compromise courses were competent in 'realistic' arithmetic, the modern courses were less favourably assessed. In many cases, compromise courses have been developed by schools. Their expressed intention is to avoid deficiencies in arithmetic and in traditional algebra to which modern courses are believed to lead. In this aim the compromise courses have succeeded.

(vi) Courses which may be described as 'traditional', 'compromise' and 'modern' occur in approximately equal proportions in schools. Much material is common to the three types. The present trend is for more schools to provide compromise courses, and for many to provide traditional courses for their less able pupils in the fourth and fifth years. The modern courses appear to make more satisfactory all round provision for the able and average pupils, but this is by no means an automatic consequence of the material. It appeared that teachers have had to think more deeply about the *raison d'être* of the material they were introducing. This has been of benefit to the general quality of the work they do.

(vii) The provision of a sufficient number of suitably qualified mathematics teachers continues to be a matter of concern. There is no indication that the needs of the schools are likely to be met in the near future. In addition, a substantial retraining programme needs to be devised for between 2500 and 4000 of the existing teachers. The greater stability of staffing of schools, which is possible in the future, has the disadvantage that there are some poorly qualified teachers occupying posts who cannot be transferred to other duties.

Other findings of this survey, which are more directly linked to the needs of commerce and industry, will now be discussed.

Links with Commerce and Industry

The School Mathematics Curriculum and the Needs of Employment

Despite all the efforts of the last decade, employers continue to give vent to their discontent with the lack of competence of their employees in arithmetic. This discontent is shared by teachers in further education. The writer has been present at several conferences in which employers have strongly attacked the influence of 'modern' mathematics.

There is little objective evidence to support their contention. There is, however, evidence enough that too many school leavers do not understand basic arithmetical concepts, and are not competent in arithmetical skills. Some of these pupils have studied 'modern' mathematics, others 'traditional' mathematics.

The concern has led to constructive action. In 1978, the Government set up a Committee of Inquiry (The Cockcroft Committee) to investigate and report upon the teaching of mathematics in schools, with particular reference to the needs of employment, higher education and adult life. The report, which is expected in the later part of 1980, may be of interest to other countries. In 1975, an Assessment of Performance Unit (APU) was set up within DES. The aim of APU is to produce, and make generally available, national profiles of various aspects of pupil performance. Its first report[6], just published, discusses its first survey, that of the performance in mathematics of 11 year old pupils in England and Wales.

A consultative document entitled *A Framework for the School Curriculum*[7], recently issued by the Government, puts forward specific proposals for a 'core' curriculum for all pupils. Mathematics is one of the subjects of this 'common core', and it is recommended that pupils should devote at least 10 per cent of their time to its study. This document also advocates policies for developing mathematics 'across the curriculum'. This implies that most teachers should be involved in the use, and therefore, in the teaching of mathematics. Another pointer to a current failure is put this way: 'Substantial attention should be given at the secondary stage to the relationship between schoolwork and preparation for working life.'

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[6] *Mathematical Development, Assessment of Performance Unit, Primary Survey Report No. 1.* Department of Education and Science, HMSO. £5. net.

[7] *A Framework for the School Curriculum.* (free) Department of Education & Science, Elizabeth House, York Rd., London SEL 7PH.

HMIs responded to this document in *A View of the Curriculum*[8]. In this they say that there is a need for a greater, and much more explicit, national consensus on what constitutes five years of secondary education up to the age of 16. They assume that a lengthy period of consultation will be required to reach it. However, it is of interest to record that the Trades Union Council (TUC) has already approved the idea of a core curriculum, and endorses the view that mathematics should be one of the compulsory subjects.

What then has been the teachers' response to the needs of the outside world? Have there been, for example, any changes in emphasis in their teaching? Some indications are given in *Aspects of Secondary Education*. On arithmetical computation and the needs of the outside world the report says that 'teachers tend to be polarised in their attitudes to arithmetical computation. A substantial number of teachers regard practice in computation as an end in itself, although the majority probably recognise the need to apply computational skills to needs arising elsewhere. Sometimes the mathematics staff within a school were divided on this important matter, and quite different emphases were given to the practice of routine skills with different classes in the same school. In other schools a consensus has been reached. But, between schools, there were wide differences in the extent to which calculations with fractions or decimals received regular practice, and in the extent to which they were practised with reference to a variety of realistic applications. It was very common for the schemes of work in the schools visited to refer to the need to relate the mathematics taught to the problems of everyday life, but the convincing realisation of this aim was much more rare.' The report also says that, whilst arithmetic was much practised, it was comparatively rare to see the skills used to build up further knowledge. And, while teachers show real concern about the lack of basic skills in arithmetic, their efforts to remedy this take the form of more and more sets of questions on fractions and decimals *without identifying the basic number deficiencies that are at the heart of the problem.*

Some of the curriculum development projects have responded to the call for relevance in mathematics teaching. The main change appears to be an increased emphasis on computation, and the associated skills, not only in arithmetic, but also in algebra. SMP, for example, is rewriting its materials with this emphasis. A particular curriculum development project of the 1960s deserves special consideration in this context. This is the Mathematics in Education and Industry (MEI) project. This project and the establishment of a 'Schools and Industry' Committee of the Mathematical Association both grew out of a two-day conference sponsored and organized in 1961 by the British Petroleum Co. Ltd. The conference brought together a number of teachers of mathematics and other interested people to study the growing shortage of mathematicians, and the kind of mathematical ability required in industry. A new GCE 'A' level curriculum resulted, and it is highly regarded by some teachers as a sound basis for the training of future mathematicians and scientists. It attracts some of the most mathematically able pupils in the country, but appears to be followed mostly by schools in or near to London.

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[8] *A View of the Curriculum.* (HMI Series: Matters for Discussion No. 11) HMSO.

Links Generated Outside the Schools

General Links

The role of CGLA in providing pre-employment courses, such as the Foundation Courses and the Numeracy Course, has already been discussed. In addition, a 'General Employment Award' has been developed as a new approach to vocational preparation for the many thousands of young people who each year enter employment without part-time release to further education.

There are other links, some of a more general nature which do not necessarily relate specifically to mathematics. The Government, through the Manpower Services Commission, created a Youth Opportunities Programme (YOP). This is a work preparation scheme with education and training tailored to the needs of school leavers. It is hoped that the scheme will cushion the growing rate of unemployment for school leavers. There have been other initiatives, notably the Industry-Education Link Projects undertaken by bodies such as the Manpower Services Commission, the Job Creation Programme, the Southern Science and Technology Forum and the Department of Industry. There has also been a successful Schools-Industry Link Scheme for Science and Technology operated in the country by the Chemical Industry since 1968. Most of these projects[9] have been concerned with links between schools and industry.

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[9] See, for example; Ruth Rees' Works: *Mathematics: Difficult or Made Difficult?* Mathematical Needs of School Leavers Entering Employment, Conference Report, I.M.A., November 1974; *The Acquisition of Mathematical Skills: The Current Trend?* Mathematical Needs of School Leavers Entering Employment II. Symposium Report No. 11, I.M.A., December 1975; *The Diagnosis of some Basic Mathematical Difficulties* Mathematics at the School/Industry Interface. Symposium Report No. 12, I.M.A., December 1975.

Mathematical Links

The Institute of Mathematics and its Applications (IMA) organized in the mid-seventies a series of three conferences[10] on the 'Mathematical Needs of School Leavers Entering Employment'. Papers were given by people from commerce and industry, and by educationalists from schools and the further education systems. These conferences provided a forum where people from these different sectors could meet and discuss their different points of view. The writer feels that the value of such conferences is in the greater understanding which can result from the discussion. Industry on the one hand appreciates better the problems of teaching mathematics in schools, the shortage of qualified mathematics teachers and so on, whilst, on the other hand, teachers realize that what they teach in the classroom really is of importance in the world beyond school.

The newly established Working Mathematics Group (WMG) provides a direct link between school mathematics and employment. WMG is a working partnership of industrialists and teachers. It produces instructional materials showing mathematics in action in the world of work. Approximately forty industries are represented in the Group and financial support is provided by several of them. Half of the members are teachers in secondary schools or in further education. Materials are produced by members working in pairs, one from industry and one from education. The Group has in mind the needs of students who have a potential interest in a career in industry or commerce or of those who have just begun such a career. Its initial concern is with the 15 to 20 year age range. This includes young people who have either left school or college and already have a job, or will soon have one.

The Group is not trying to provide a complete course in applicable mathematics, but, rather, to build bridges capable of carrying traffic in both directions across the interface between employment and mathematics education. Its first materials are to be published in June, 1980. These are concerned with finance: *Understanding Interest.* Other materials in preparation involve mathematics related to engineering, aircraft design, transport and so on.

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[10] Manpower Services Commission, Job Creation Programme. *Industry-Education Link Project, An Exploration of the Interface between School and Employers, Final Report – August 1978.* Information and Enquiries: The Director, Southern Science & Technology Forum, Southampton SO9 5NH. *Southern Science & Technology Forum. Autumn Journal, A Start in Life, Incorporating Annual Report 1977.* Information and Enquiries: The Director, Southern Science & Technology Forum, Southampton SO9 5NH. *The Schools-Industry Link Scheme*, prepared by Dr. B.A. Haines, Information Officer, Schools Information Centre on the Chemical Industry, in collaboration with M.J. Tomlinson, Head of Chemistry Department, Ashby-de-la-Zouch Grammar School, Leicestershire. Published by the Schools-Industry Link Scheme Advisory Committee, Alembic House, 93 Albert Embankment, London SE1 7TU. *Case Studies of Industry/*

Links and Research Studies

Research studies are of two kinds. One is concerned essentially with 'fact finding', so that rational decisions can, if necessary, be taken or so that further research or development may be promoted. Another kind of research is concerned with more fundamental problems of learning and teaching. An example of the first kind is the Schools Council study *The Mathematical Requirements of Young Entrants to Employment.*[11]. This grew out of a survey of mathematics projects involving education and employment, prepared by Professor D. Bailey of Bath University[12].

The Schools Council study has revealed many kinds of local liaison between schools and industry. For instance, in some places, employers are devising various mathematical tests to select school leavers for employment. And there are examples of teachers modifying their school syllabus to meet the apparent requirements of local industry. A particular characteristic of such local groups is the dependence of the liaison on individual enthusiasm. The departure of one participant may lead to a collapse of the work. There is evidently a need to co-ordinate such projects, and for some of them to benefit from a knowledge of the good practice of others.

Another study is that carried out by T. Crudge, a B.P. Research Fellow in Scotland, *Mathematics in the Community*[13]. This study has been investigating the applications of mathematics in employment in order to provide teachers with realistic examples of problem solving which they can use in the classroom.

The mathematical knowledge and skills required by pupils entering the engineering industry at the age of 16 were investigated by A. Fitzgerald[14]. Another study, involving several industries, has been carried out by Mrs. C. Knox at Sheffield. These studies (and others) have demonstrated that employers essentially require school leavers to be competent in basic arithmetic with a grasp of the concepts and a

Education Links, A Selection of Local Initiatives to Encourage a Better Understanding of Industry. Industry/Education Unit, Department of Industry, Ashdown House, 123 Victoria Street, London SW1E 4RB.

[11] *The Mathematical Requirements of Young Entrants to Employment.* Schools Council Publication (in press) 160, Great Portland St., London W1.

[12] Professor D.E. Bailey. *A Survey of Mathematics Projects involving Education and Employment.* University of Bath, School of Mathematics, October 1978.

[13] T. Crudge. *Mathematics in the Community, B.P. Research Study.* Paisley College of Technology, Scotland.

[14] A. Fitzgerald, Mathematical Knowledge and Skills required by Pupils entering Industry at the 16+ level, *Mathematics in School*, May 1976, School of Education, University of Birmingham.

mastery of the skills. Similar studies of the requirements of various kinds of employment have recently been commissioned by the Cockcroft Committee.

Studies of a more fundamental nature into the learning and teaching of mathematics were pioneered in the early 1970s by the Mathematics Education Group at Brunel University.[15] These studies initially investigated the mathematical performance of students at colleges, schools, teacher-training institutions and at a university. The aims have been to diagnose learning difficulties in mathematics and to investigate the reasons for such difficulties.

The studies originated in further education, where mathematics is applied in the various contexts of vocational training. Teachers in colleges of further education took it for granted that they could rely upon their students leaving school with a competence in basic mathematical concepts and skills. This is not the case. Some of the findings are as follows:

a. Many pupils leave school with a poor understanding of basic mathematical concepts, and they possess few of the associated skills.

b. Some failures in understanding and certain types of errors appear to be common among all kinds of students, including even undergraduate engineers.

c. These failures and errors have their roots in primary schooling. They persist in secondary schooling and continue into further and higher education.

d. They are shared by primary school teacher trainees and by others not specializing in mathematics. This indicates a learning-teaching cycle.

e. Experienced teachers of mathematics often do not appreciate the nature of these failures in understanding, and underestimate the extent of their students' difficulties.

The development programme resulting from the research includes discussing with teachers possible teaching strategies and teaching styles which would avoid the misunderstandings which give rise to the failures and errors to which students are most prone. Tests are also being designed which will make it possible to diagnose the existence of misunderstandings in the minds of learners when they pass from primary school into secondary school and from secondary school into further education and for teachers in training.

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[15] Details of publications may be obtained from: Mathematics Education Group, Department of Education, Brunel University, Uxbridge, Middx.

These studies have highlighted the fact that mathematics education is a continuous process. The kind of learning that takes place at school, first at the primary then at the secondary level, does affect learning at later stages of training, in employment and life.

Links through Mathematics: Some Examples.

The following calculations, amongst many others, were designed by the writer during investigations of the mathematical difficulties experienced by school pupils and further education students. They illustrate how precisely the *same calculation* can be embedded in a variety of quite *different contexts.*

Example one

(i) $8 \div 0.16$ is . . . .

(ii) If $0.16x = 8$ then x is . . . .

(iii) Given that Resistance = $\dfrac{\text{Potential Difference}}{\text{Current}}$, an appliance carrying a current of 0.16 amperes with a potential difference of 8 volts has a resistance of . . . . . .

The first context is a 'pure' calculation, the second context is algebraic, and the third is vocational.

A pupil at school will encounter the calculation in the first two contexts. If the pupil studies physics the problem will also be met in the third context at school: otherwise it will be met in electrical engineering studies in further education.

Example two

(i) $0.000011 \times 5 \times 20$ is . . . . . .

(ii) For an increase in temperature of 1°C a 1 mm length of steel will expand 0.000011mm. For an increase in temperature of 5°C, a 20 mm length of steel will expand by . . . . .

Example three

(i) If $\dfrac{1}{R} = \dfrac{1}{2} + \dfrac{1}{6}$ then R is . . . . .

(ii) The effective resistance R of a resistance R_1 connected in parallel with a resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

if R_1 is 2 ohms and R_2 is 6 ohms then R is . . . .

Example four

(i) $1 - (\frac{1}{3} + \frac{1}{4} + \frac{1}{8})$ is . . . .

(ii) A government report announced that the 'average' family (a couple with two children) spends one-third of its income on housing, one-quarter of its income on food and one-eighth on heating. The fraction left for everything else is . . . . . .

It will be seen that the applications of the calculations in the first three examples are given in technical contexts, whilst that in the fourth is in a social context. Many contexts could be chosen from other school subjects as well as contexts from employment and living. The Open University is at present devising a course called 'Mathematics across the curriculum'.

The report *Aspects of Secondary Education in England* points out that teachers tend to give 'lip service' only to 'realistic' arithmetic. It would be a time-consuming task, and not an easy one, for teachers to devise their own sets of realistic examples. So the work of a group like the Working Mathematics Group offers a valuable support to teachers.

The writer would emphasize, however, that, whatever the contexts used, teachers must always be alert to the real nature of the mathematical difficulties of their pupils/students. Putting a calculation in a realistic context may motivate, but it does not remove the mathematical difficulty.

The research studies showed that problems, such as those given in examples one and two, revealed that school pupils and further education students did not understand the effects of the operators '×' and '÷' on small numbers, that is on numbers less than unity. Problems, such as those in example three, showed that a common difficulty was typified by

$$\frac{1}{x} = \frac{3}{4}$$

School pupils and further education students 'did not know what to do with x on the bottom' (if x was not a small whole number).

The student's perception of a problem does not always match that of the teacher! Language laboratory recording of students 'talking their way through' to the solution of problems, showed that there were several perceptions of a problem. Consider, for example the case of

$$\frac{1}{x} = \frac{3}{4}$$

Some school pupils used the concept of equivalent fractions, but had no skill applying the concept. Some were thoroughly confused because 'x is on the bottom'. More mathematically able pupils, and older students, saw an equation and 'multiplied both sides by'. Further education technician students used 'cross-multiplication', whilst many craft students were thoroughly confused like the pupils. Scientists said 'it's a reciprocal'. A few very able students said 'turn both sides upside down'.

Language laboratory recordings also reveal, in a way that written work does not disclose, the 'blockages' which occur when small numbers are multiplied together. Several calculations such as 0.9 × 0.9, 0.8 × 0.8, 0.7 × 0.7, 0.4 × 0.4, may generate correct solutions. It is only when the same (but wrong!) 'rule' is applied to 0.3 × 0.3 to give 0.9 that the failure in understanding is revealed. Students are multiplying the small numbers as if they were whole ones and then 'sticking in a decimal point' without a real understanding of place value, and the fact that when small numbers multiplied together the result is an even small number. This is contrary to the student's inbuilt expectation that the 'numbers get bigger when you multiply'.

Such calculations look deceptively simple, yet, if there is initial conceptual confusion, there is little hope of an application of a correct concept to achieve solution. The applications of 'the inverse' and of numbers less than unity are many and varied. The inverse is used in many scientific applications, in speed, distance and time problems and in other branches of mathematics. It is therefore important that the teacher should be aware of the nature of the blockages associated with computational concepts and skills, because embedding the computation in a realistic context can create additional blockages. The writer has found that pupils and students first have difficulty selecting the relevant data, then the relevant operators and finally the correct sequence to solution. Several craft students expressed it thus; 'we are motivated by the contexts relevant to our work, but all that 'junk' (i.e. the words) confuses us. It's alright when the teacher does it, but we do not know where to start when we are on our own'.

One of the secrets of effective teaching must surely lie in selecting examples to practise which reinforce understanding. Richard R. Skemp[16] points out that if examples are 'well chosen, with plenty of variability and increasing noise, there should be no chance of this kind of practice degenerating into rote memorising'. The research studies have shown that what pupils learn at school very much affects their success in further education, employment and living. Mathematics education should be thought of as a continuous process from school to commerce, industry and to society in general.

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[16] Richard R. Skemp. *Intelligence, Learning and Action.* Wiley & Sons, 1979.

Conclusion

It will now be apparent that, in England, there are many attempts to forge links between school mathematics and employment. The escalating number of 'link projects' indicates a desire on the part of both schools and industry to try to come together in a better understanding than has hitherto existed. Any overview made of the scene and any attempt to draw conclusions cannot but be subjective. In this sense the following observations are offered. Such surveys as have been made, and such research studies as have been carried out reveal that some links are sound and lead to good practice, whilst others are ineffective and lead to bad practice. There is a danger that the time and effort involved in generating and maintaining links may sap the energies of teachers and divert their attention from the fundamental problems of teaching and learning. Studies show that what most school leavers lack is a proper understanding of basic mathematical concepts and of the associated skills, and an ability to apply them to a practical problem. But these things are precisely what industry, commerce and adult life generally require. The enormous efforts that have been put into changing the content of the curriculum do not appear to have led to any significant improvement in these aspects of mathematical ability.

The fundamental problems of the teacher are surely those associated with how pupils *learn* mathematics, irrespective of its label, 'modern', 'traditional' or 'compromise', and irrespective of its content. The teaching of arithmetical concepts and skills in such a way that pupils learn to understand the concepts, develop skills, and acquire an ability to apply them appears to present problems that are of international concern. There has been a tendency on the part of mathematicians to assume that all that is involved in computational mathematics is routine skills. They have underestimated the depth of the conceptual understanding which average pupils must reach before they are able to apply their knowledge as a matter of routine. Teachers, for their part, seem very happy to engage their pupils in discussion of descriptive mathematics such as 'shape', 'tesselations' etc. But very seldom do they discuss problems which involve computation. It is, however, the abstraction of the symbolism which creates the difficulties in computational work. This feature should merit even greater discussion. The writer would suggest, as a result of many years experience both in the classroom and of research studies, that educational systems should pay much more attention to the following:

1. Teachers should be made aware of the *real* nature of the pupils' difficulties. Research has shown that these difficulties can be quite subtle. Teachers do not usually appreciate the nature of

the difficulties. They tend therefore to underestimate the extent of such difficulties.

2. Many teachers suffer often unwittingly from the same fundamental difficulties as their pupils. Until these difficulties are eradicated the vicious cycle of inadequate teaching and inadequate learning cannot be broken.
3. The teaching of the concepts and of the skills should be *explicit.* Too much is thought to be commonplace, and not worth pointing out.
4. The teaching should aim for *understanding* by the pupil.
5. Understanding requires a better appreciation of the importance of *language.* This requires endless *practice.* Language should be practised *orally.* And *written* forms of language should be discussed and unravelled orally, until comprehension is reached and a decision on action to be taken can be made. (The writer has been trying to persuade teachers to use the language laboratory for mathematics.)
6. Mathematics should be employed more effectively 'across the curriculum' in schools. And examples of the applications of mathematics in the world outside school should be brought into schools.

If a pupil leaves school with a mastery of the concepts of number and of the skills associated with the handling of numbers, then employers and teachers in further education and other people involved in training will be better able to concentrate on the applications to the context of a specific job.

Finally, the writer believes that a very important link has to be forged with society. This is a *general awareness* of the significant role that mathematics plays in society and in unravelling the secrets of nature and of the universe. Such a general awareness may help towards a better appreciation of mathematics by pupils at school. It could be achieved by the appropriate use of the current mass media of communication. Perhaps an international 'Public Relations Exercise' for the international community should be devised and launched by mathematicians!

Herbert Vockenberg

Goals of Mathematics as a Reflection of the Requirements of Production[1]

The principle of connecting school with life is one of the fundamental principles of Marxist-Leninist education and of educational policy in the countries of real socialism. For its implementation, polytechnical education and training is of central significance.[2]

We understand polytechnical education and training to be 'a specific orientation of general education so as to qualify the young generation for productive work, and for mastering the tasks associated with science, production and technology'.[3] Mathematics Education, which is imparted in our schools as a component of general education, also requires such an orientation for similar reasons. It is the purpose of this article to describe how this orientation is accomplished for the mathematics teaching given in the ten-year general polytechnical secondary school, the integrated normal school for all children and young people in the German Democratic Republic. Mathematics teaching in the stage leading to matriculation for higher education (*Abiturstufe*) – a stage followed by only some of the school leavers from the ten-year secondary school – will not be dealt with here.

Mathematical education in the ten-year general polytechnical secondary school is acquired mainly, but not exclusively, in mathematics teaching. Consequently, its objectives, content, methods, means, and forms of organization must be determined in such a way that it can both meet the requirements of polytechnical education and training (as a central task to be resolved by the socialist school) and at the same time implement its other objectives and tasks.[4] What this requires, in detail, is essentially determined by the role of mathematics in production, in technology and in other sciences, and by the demands which result from this for the mathematics education of the working people.

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[1] Abridged and edited version of the author's paper "Polytechnische Bildung und Erziehung im Mathematikunterricht der allgemeinbildenden polytechnischen Oberschule der DDR" [Polytechnical Education and Training in Mathematics Teaching in the General Polytechnical Secondary School of the German Democratic Republic], in: *Polytechnische Bildung und Erziehung im mathematischen und naturwissenschaftlichen Unterricht in der allgemeinbildenden Schule der DDR und der USSR* [Polytechnical Education and Training in Mathematics Teaching and in Natural Science Teaching in the General School in the German Democratic Republic and in the USSR]. Worked out by a group of authors headed by H. Frankiewicz, Volk und Wissen Volkseigener Verlag, Berlin 1979.

[2] See Dietrich, G., Küstner, R. and Voigt, H.: Zur Demokratisierung der Bildung und Erziehung in der DDR [ON the Democratization of Education and Training in the German Democratic Republic]. (unpublished).

[3] Frankiewicz, H.: Die polytechnische Bildung als Allgemeinbildung [Polytechnical Education as General Education]. *Pädagogik,* No. 12, 1972, p. 1119.

The Role of Mathematics in Production, Technology and other Sciences[5]

As a result of the scientific-technological revolution, the significance of mathematics for production, technology and other sciences is continuously growing[6]. In addition, it is penetrating more and more fields of human activity. Mathematics is, for example, playing a growing role in the planning and management of socialist production, in the comprehensive protection of the socialist community of states, and in many fields of research and development.

Mathematics has developed from practical requirements: those of land surveying, of time measurement, of navigation, etc.; and 'as a means of reflecting certain aspects of the world surrounding us'[7]. Due to the internal problems it poses, as well as the new demands of social and other sciences, it continues to develop. Today it comprises a relatively independent system of theories about existing objective structures or possible structures conceived on a very high level of abstraction. It also embraces general and special methods which are used to acquire knowledge within other fields of knowledge and to ensure the planned and purposeful change of nature and of society. Mathematical theories, methods and processes are becoming more and more effective in technical research and development, and, as a result, mathematics is becoming a productive force. This is because available and planned production processes can be worked through with mathematical models, and can even be partially optimated. This also applies to problems of distribution, transport, communication etc. In addition, there are some technologies which only become possible by applying mathematical findings and methods. Such applications of mathematics can considerably decrease the cost of a unit of production or of transport. In this sense, mathematics is a scientific basis for almost any planned process in the fields of research, development, production, and distribution, and for the solution of tasks and problems in many other spheres of social practice. In this broad field, electronic data processing plays a special part. Electronic equipment makes it possible to master the processing of data (which crop up on a large scale in the various tasks mentioned) by means of elementary mathematical operations. The processing is done with such rapidity and with so little labour that it is becoming worthwhile for more and more institutions to use electronic data processing. Computers and micro processors also make it possible to apply modern technologies in production – technologies which could not be applied without this equipment.[8]

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[4] Methodik Mathematik unterricht [Methods of Mathematics Teaching]. Worked out by a group of authors headed by W. Walsch and K. Weber. Volk und Wissen Volkseigener Verlag, Berlin 1975, pp. 13.

[5] Weber, K., H. Vockenberg: Mathematikunterricht und polytechnische Bildung – Ein Beitrag zur Standpunktbildung (Mathematics Teaching and Polytechnical Education – a Contribution for Forming an Opinion). In: *Jahrbuch der APW der DDR*, Berlin 1972, pp. 358

[6] Hager, K.: Sozialismus und wissenschaftlich-technische Revolution (Socialism and Scientific-technological Revolution). In: *Neues Deutschland*, 22 June 1972, p. 4.

[7] Gnedenko, B.V.: Abbildtheorie und Mathematik [Reflection Theory and Mathematics]. In: *Mathematik in der Schule*, No. 9, 1977, p. 449.

It is also a characteristic of the application of mathematics in social practice and in other sciences that there are, in general, no unequivocal classifications between the individual disciplines within mathematics and their fields of application. This is the case with such natural sciences as the physics of semi-conductors, the chemistry of blast furnaces, or agricultural chemistry. The reason is that the abstract structures and the statement of problems investigated in individual fields of mathematics appear in very different spheres of human activity, and also in very specific concrete manifestations. Therefore, even relatively simple and special mathematical methods can frequently be applied in very different social fields. The broad field of applying linear optimization, and statistical quality control are such examples.

But, also, very abstract theories, such as mathematical logic or Boolean algebra have been applied in a very broad way in many fields of social practice as a consequence of the mediation of electronic data processing equipment and cybernetic systems.

The Contribution of Mathematics Teaching to Polytechnical Education and Training

In a socialist society, such a comprehensive utilization of mathematics cannot be the individual concern of specialists (mathematicians, scientists of other disciplines, technologists, economists etc.). Rather, it is desirable that working people should use mathematical findings and methods directly in their work, or use them indirectly with relevant equipment. For this, mathematics teaching at the general school must create the essential foundations and preconditions.

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[8] The Realization of the Resolutions of the 9th Party Congress of the Socialist Unity Party of Germany in the Field of Electrical Engineering and Electronics, Resolution of the 6th Session of the Central Committee of the SUP, *Neues Deutschland*, 27 June 1977, p.4.

To do so is not easy. The application of mathematical methods and processes in social practice are extremely specialized and efficient. And they are more and more connected with the use of expensive equipment. The introduction of these mathematical applications into school teaching would lead to a confusing diversity of subject-matter, which would never be mastered anyway, and which would be very difficult to arrange in a systematic manner. More importantly, the advantage of mathematical methods and processes (their broad applicability) would not be made sufficiently clear. To deal with specific applications of mathematics cannot be the main mode of mathematics teaching in the general school. Nor can mathematics teaching concentrate on the direct solution of 'topical' problems using mathematical techniques, because this would require comprehensive mathematical knowledge and skills on the part of the pupils which they have yet to acquire.

However, as in any other science, there are also in mathematics – and especially so in mathematics – fundamental elements which can be regarded as relatively stable, and which are at the same time the basis of any further mathematical education, including specialized mathematics education for vocations. This will remain so in the foreseeable future. These elementary mathematical items must remain basic to mathematics teaching in the general school, not only for educational reasons, but also for reasons of preparation for those mathematical requirements which will crop up in vocational and further training, as well as later – on the job and in the vocation itself.

Nowadays we make the assumption that this training can only be achieved by mathematical education orientated towards those fundamental items which are needed both for on-going education and for use in other school subjects and in life. The experience of the last ten years shows that this requires an orientation of mathematics teaching both towards the systematics and structure of the subject as well as towards typical applications to everyday problems and to other school subjects. The significance of mathematical skills is emphasized by requiring comprehension of their applications and by demanding the ability to apply, unaided, mathematical knowledge to circumstances, problems and tasks in production, technology and science. This statement does not affect our fundamental position that exact mathematical knowledge is an inalienable precondition for the development of mathematical skills. But it must be realized that the 'individual value' of mathematical skills is relatively low. It is the *ability to apply* this knowledge to mathematical and non-mathematical circumstances and tasks which is the hallmark of fruitful mathematics education and training, and particularly for polytechnical education and training. Therefore, mathematics teaching in our general schools is orientated, not so much towards imparting comprehensive mathematical knowledge, but towards using to the maximum the mathematical knowledge that is imparted.

The young generation should also be prepared to cope with an increasing application of mathematical findings and methods in their later vocational activities. This means providing them with lasting and fundamental mathematical skills – skills which are solid, well-founded, extendable, and applicable in a high measure, and forming a basis for learning all their lives.

This does not, by a long way, exhaust the significance of mathematics teaching in polytechnical education and training. There is also the need to connect the development of the above-mentioned knowledge and skills with the development of general mental abilities, with qualities of thinking and with intellectual habits, which the pupils need not only at school but also in their future jobs. A big part is played in this by disposability, self-reliance and creative work.

Finally, preparation for work and vocation also necessitates the use of the *educational potential* of the subject-matter and the processes contained in mathematics teaching. The general aspects of mathematics teaching emphasized above (with regard to content and methods) are not only essential for the realization of polytechnical education and training. The development of fundamental mathematical knowledge and skills is also significant for objectives in the field of mental education. This multivalence of objectives in mathematics teaching is also valid for the general school, and is thus valid for compulsory education generally.

The Implementation of Polytechnical Education and Training in the Framework of the Objectives and Contents of Mathematics Teaching

The mathematics course in the curriculum is orientated towards the content of the science of mathematics, on its method and processes, on its ways of working and thinking, and its educational usefulness. The essential objectives of mathematics teaching comprise the acquisition of fundamental mathematical knowledge and skills, with special regard for their applicability to mathematical and non-mathematical tasks and of statements of problems. The implementation of these objectives forms a basis for deeper penetration into the science of mathematics and for understanding and the development of elementary abilities and skills in the application of mathematical methods to life. In our opinion, the complexes listed below of the mathematical knowledge and skills to be acquired by the pupils are of special significance for polytechnics.

Working with Numbers and Quantities

Numbers, relations between numbers and the basic arithmetical operations associated with them, and with quantities based on them, are indispensable for the quantitative understanding and description of reality. For this reason, they are central to the subject-matter, and indispensable as a foundation of all our teaching in mathematics. Consequently, solid and lasting skills in using, and calculating with, numbers and quantitites are an indispensable precondition and a central objective of polytechnical education and training.

It would be incorrect to emphasize only those elements of knowledge and skills whose polytechnical relevance is immediately obvious, like working within certain technical tolerances, and problems of precision of measurement and accuracy of calculation. It is, rather, a question of including those elements of subject-matter which must be mastered as a necessary pre-condition for the realization of polytechnical education and training, both in the subject of mathematics as well as in other subjects. Accordingly, the relevant questions are discussed below, following the structure of the mathematics curriculum in the general school.

The knowledge and skills in numbers and calculating with them is of fundamental importance. Particularly the algorithms for mental calculations and especially those for written calculations must be mastered reliably. From the polytechnical aspect, it is also important to pay special attention to problems stated in words and problems from practice, as is done in the primary stage and in the first forms of the medium stage. These particularly simple forms of mathematical modelling of given facts and of problems are of considerable significance for polytechnical education and training, because they correspond largely – or, at least, the come very close to – typical applications of mathematical processes in real life. The transition from the perceived facts, or from the factual problem to a mathematical model, the operating within this model (solving the relevant mathematical problem) and the factual interpretation of the solution found to the mathematical problem comprise that three-fold process which must be regarded as characteristic of all applications of mathematics to life.

In order to solve the problems mentioned, so as to improve further polytechnical education and training, the following are in need of emphasis.[9]

- Special emphasis needs to be placed on mastering the fundamental knowledge and skills of working with numbers and quantities, as are applied to many processes, especially in mental and written calculations.
- To this end it is necessary also to impart a certain minimum of facts and formulas to be reliably memorized. The teacher should see to it that the pupils use these consistently and self-reliantly in solving problems. This is an indispensable precondition for rapid, efficient and reliable work when no aids or storage are available, as is often the case in every-day life and in later vocational work.
- The use of aids for calculation (numerical tables, slide rules, pocket electronic calculators) and storage (numerical tables, collections of formulae) must, on the one hand, remain restricted to such cases where it is sensible to employ them, but must, on the other hand, be planned and used more systematically and reliably than at present.
- It is essential to develop sufficiently exact conceptions of the order of magnitude of the given or calculated values (numbers and quantities) which are used in every-day life, in polytechnical instruction and in other school subjects.
- Special attention should be paid to developing skills in making 'rough calculations'. The following are especially important:
  - rapidly finding the approximate result (this often will suffice to answer the problem stated);
  - determining the order of magnitude as an auxiliary problem (e.g. in using the slide rule) or checking a result calculated in a different way (in the sense of checking whether a necessary condition for the correctness of the result has been fulfilled).
- It is especially important to get the pupils into the habit of checking, and of finding their own ways of checking the answers to as many problems as they have to solve.
- Furthermore, to acquire in measurements and calculations an understanding of necessary and adequate accuracy.

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[9] Sotschek, P. Zur Verwirklichung der polytechnischen Bilding und Erziehung im Mathematikunterricht beim Arbeiten mit Zahlen and Grössen sowie mit Gleichungen, Ungleichungen und Funktionen [On the Realization of Polytechnical Education and Training in Mathematics Teaching in Working with Numbers and Quantities and with Equations, Inequations and Functions]. In: *Polytechnishe Bildung und Erziehung im mathematischen und naturwissenschaftlichen Unterricht in der allgemeinbildenen Schule der DDR und der UdSSR* [Polytechnical Education and Training in Mathematics Teaching and in Natural Science Teaching in the General School of the German Democratic Republic and the USSR]. loc. cit. pp. 62.

- A deepened understanding of central mathematical and theoretical terms used in 'Working with Numbers and Quantities', such as 'Operations' and 'Algorithms', as well as developing algorithmic ways of thinking and working in general; this includes enabling the children to use algorithms in practice.

Through the above-mentioned emphases, it is primarily, but not exclusively, the concern of mathematics teaching to ensure mastery of the complex of subject-matter in 'Working with Numbers and Quantities'. It is also important that the pupils' mathematical knowledge and skills are applied to other subjects. This by no means concerns only the natural science subjects in the general school. It concerns at least as much polytechnical instruction, because mathematical knowledge and skills can be applied in a practical way, beginning with handicraft lessons in the primary stage and going on to productive work in the higher forms. It should be consistently used for the realization of the objectives pursued, by organizing close co-operation between the subject teachers and the instructors concerned. This co-operation can be carried through with combined methods and division of labour.

Working with Equations, Inequations and Functions

Knowledge about equations and inequations (with at least one variable) and about functions is also of great importance in polytechnical education and training. Mainly required is skill in stating and solving equations and inequations and in examining, representing and applying functions. These topics are essential mathematical bases for understanding, and for giving descriptions of phenomena in nature, processes in technology, in production, and in other spheres of the national economy and in society. They are also a means to performing tasks and solving problems which arise in those fields. It must not be overlooked, however, that types and classes of equations, inequations and functions dealt with in the general school are of such a simple structure that only simply structured relationships can be illustrated, and only simple problems arising therefrom can be solved. This knowledge and these skills are, however, an essential precondition for understanding the complicated relationships that are investigated by other means. It is therefore of primary importance to develop a reliable and lasting grasp of 'Equations, Inequations and Functions' and not to get stuck in the mire of 'technical details'.

In order to help the pupils to apply this knowledge and these skills in other fields, it is necessary to deal with simple applications. It is, however, necessary to be circumspect about problems stated in words, particularly those which lead to linear or quadratic equations or to systems of linear equations, for these are often unnatural, and of little polytechnical substance. In such problems, the content has often to be very much simplified, and the statement of the problem is often alien to practical experience. This should be made clear to the pupils. But, as long as better problems are not available, they can and must be dealt with, because the pupils have to practise intensively the mathematical representation of simple and simplified relationships taken from life or from the sciences. Due however to the wide differentiation of circumstances and requirements – particularly in the pupils' productive work – these opportunities of application are specifically connected with the particular job. They can hardly, therefore, be built into the mathematics curriculum and into the teaching materials published. In order to deal with them in the mathematics lessons, thorough analyses of the pupils' places of work are necessary. In places where this has already been done, the results should be used by the mathematics teacher in order to relate his lessons more closely to real life and consolidate the mathematical knowledge and skills necessary to meet practical requirements.

For the better realization of polytechnical education and training in the subject-matter dealt with here, the following aspects need emphasis[10].

- It is necessary to include the 'formulae' of physics, technology, etc. in mathematics teaching. These formulae should be looked at from a mathematical standpoint. They should be seen for what they are: as equations, inequations, or as the depiction of a function. They should be manipulated mathematically in the usual way. The results should then be interpreted in context. Analogous methods should be applied to other school subjects.
- It is extremely important to work consistently, so as to develop the abilities and the skills necessary to represent the simple facts and the problems which arise in physics, technology, production, the transport system, military technology etc. in equations, inequations, tables, or diagrams, and to work with these as described above.
- It is of no less importance to develop the ability to interpret the given mathematical equations, inequations and functions in a practical manner. That is to say that the pupils should learn to be able to give the possible circumstances or to state the problems of which these equations are 'models'.
- Finally, it is necessary for students to apply mathematics to problems in the natural sciences and technology which should provide a deeper understanding of mathematics.

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[10] See again, Sotschek, op. cit.

In general the curriculum offers good opportunities to fulfil the above demands. The task still to be solved is to recognize the opportunities, and to fit them organically into the teaching process.

Working with Geometrical Terms, Statements and Processes

It is important for polytechnical education and training that the pupils should learn about the geometrical and the topological relations and properties, of plane figures, as worded in elementary geometrical definitions and theorems. They should also acquire skill in goemetrical drawing and in constructing geometrical figures. They need to be able to depict simple three-dimensional shapes using different projections. They must be able to calculate areas and volumes. This is because the methods and processes of geometrical depiction and construction are a basis for technical design and development, especially for technical drawings and diagrams. Geometrical representations play an extraordinary role as information carriers in technology and production. The knowledge and skills acquired in the geometrical course of the general school provide a theoretical basis for working out and for understanding technical documents.

The main emphases to be placed in geometry teaching are:

- Special attention should be paid to the development of the pupils' ability to think in three dimensions. In this regard, the mathematics curriculum, with its introduction to elementary terms, theorems, methods and processes of descriptive geometry, and their application to simple geometrical bodies, offers what is absolutely necessary. But studies of school work often show a one-sided preference for constructions of plane projections of a solid figure. And since the understanding of a technical drawing, and indeed of any picture, involves the opposite procedure (the interpretation of a plane drawing as a solid object), the relevant exercises of interpretation should not on any account be neglected.

Specialized Language and Symbols of Mathematics

The polytechnical importance of the knowledge and skills imparted in mathematics lessons would be incomplete if the following were not considered: the need to acquire, step by step, the essential elements of the specialized language and symbols of mathematics; the need to ensure that the whole of the subject-matter is permeated with the theory of sets and of logic. These basic objectives and content of mathematics teaching, which were introduced into the curriculum now operating in the ten-year secondary school of the German Democratic Republic, reduced the gap between the content of mathematics teaching and mathematics as a science. They are of great polytechnical value. By way of illustration, it is sufficient to remember the efficiency of mathematical symbols in describing reality, and of the role of mathematical logic in computer engineering and in other sciences. These rather abstract, factual and methodological elements of mathematics teaching make it possible to enhance the quality of polytechnical education and training in mathematics teaching. This is because:

- they achieve a more abstract, a deeper, a more comprehensive and a more precise reflection and description of the natural and technical environment of man than would be possible without;
- they make it possible to solve by mathematical means other problems from many social spheres, especially from science, technology and production;
- they allow a better structuring of the mathematical content and a better emphasis of the sustaining terms, statements, thinking and working processes.

But it is also clear that more intensive work with variables, with the elements of mathematical logic and with the theory of sets, are of direct importance to polytechnical education – because these aspects of the subject are being used increasingly in engineering and technology. But it must not be overlooked that completely satisfactory solutions have not yet been found to the teaching of these topics in the present mathematics curriculum of the ten-year secondary school.

The Polytechnical Significance of Different School Societies according to a Skeleton Programme

According to a skeleton programme in forms 9 and 10, school societies are of fundamental polytechnical significance for several reasons. It is one of their most important functions to react to new social requirements in education. This they can do quicker than is possible for normal school work. Furthermore, they enable pupils to acquire knowledge in specialized fields according to their special inclinations and interests, beyond that imparted by compulsory general education. This can be and should be equally important for their choice of vocation, for the development of their special interests and for the development of their mental and practical abilities, as well as generating socially valuable convictions, and good personal attitudes.

In the school society for mathematics, the issues of applying mathematics in life play a significant part. This is evident from the skeleton programmes, 'Electronic Data Processing and Computer Engineering' and 'Practical Mathematics', which deal with developments of increasingly practical importance, which have not yet been included in compulsory general education. The skeleton programme, 'Elementary Statistics', too, is very practically orientated. And, whereas 'Working with Sets' is devoted to a main mathematical topic, it deals with it in a very practical way, including the 'every-day experiences' of the pupils.

The Polytechnical Significance of the Objective of Mathematics Lessons Touching on Other Subjects

The contribution which mathematics teaching has to make to polytechnical education and training is not confined to the mathematical knowledge and skills that are imparted. As one of the key subjects in the curriculum, which absorbs about 20 per cent of the time allocated, mathematics teaching makes a significant contribution to the realization of the general objectives of the general school.

What is important to the polytechnical aspect of this contribution can be explained in two important objectives: the development of mental abilities (of scientific ways of thinking and working) and of education for work.

In the matter of *developing mental abilities*, our assumption is that these develop in and through activities which need the same embryonic abilities in the first place. It is therefore advisable to compile those general mental processes which are characteristic of mathematics, and which are performed on mathematical or mathematizable objects by applying mathematical methods and processes. Without claiming to have compiled a complete list, and without giving an order of precedence, these mental processes are:

- giving, and defining terms with a relatively high degree of abstraction or generalization;
- making statements (by reduction and deduction);
- checking statements and statements of relationships as to their truth by using the formal conclusion and inference from the content[11];
- obtaining rules for solving and applying classes of problems;
- using heuristic methods to solve problems (the strategy of working forwards and backwards, the strategy of reduction etc);
- using specific methods of checking which are typical of the subject;
- utilizing appropriate material or other aids.

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[11] Weber, K.: Zur Realisierung des polytechnischen Prinzips im Mathematikunterricht [On the Realization of the Polytechnical Principle in Mathematics Teaching]. *Mathematik in der Schule*, No. 10, 1975. pp. 545.

These activities, and the use of precise and clearcut language and the symbols of mathematics, make it possible to develop abilities and reasoning qualities whose importance extends far beyond mathematics teaching. If they are correctly used in mathematics teaching, they serve to develop the following characteristics of personality:

- Thinking and reasoning based on clarity and precision, and a form of oral and written expression which is characterized by brevity and preciseness;
- Complete, and conclusive reasoning and argument;
- The development of thinking in variants;
- Intellectual versatility and creative ability;
- The ability to differentiate between the essential and the non-essential;
- Positive attitude to solving problems on the basis of developed abilities in the self-reliant application of algorithmic processes and heuristic methods;
- Self-reliance in the selection and appropriate use of aids;
- The development of the ability to acquire knowledge in private study, particularly to acquire mathematical knowledge and skill which is an essential precondition for continuing to learn all through life;
- Self-critical and critical ability to assess and check the results of work.

Without further explanation, it is clear that such characteristics of personal development are extremely valuable for work and the vocation and also for other spheres of human activity.

Education for work is not possible without the pupils' direct participation in production for 'how to work is only learnt through work'[12]

In this respect, mathematics teaching can and must make an essential contribution. We fully agree with Frankiewicz when he states that the education for work begins in lessons[13], and we think that it must be continued there consistently and purposefully. This is due to the fact that learning and working, being fundamental forms of human activity, have essential characteristics in common. Both are directed towards an objective and a seriousness of purpose[14], and they cannot often be isolated from each other in their concrete forms. This also becomes evident in many situations during lessons. Whether the pupils are listening to the explanation of a problem, or to a proof – by the teacher or a fellow pupil – whether they are working together (with the whole class or only a part of the class) to solve a problem, or whether every pupil is solving a mathematical problem for himself – a complex activity for the pupil is always present, an activity which is planned beforehand by the teacher with certain objectives and is guided by him more or less intensively. In such activity – as in work – the whole personality of the pupil is taking part –his knowledge and skills, his attitudes and convictions, his motives and his will.

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[12] Dietzel, K.: Stand und Aufgaben des polytechnischen Unterrichts bei der weiteren inhaltlichen Ausgestaltung der Oberschule (The Level Reached and the Tasks to Be Solved in Polytechnical Teaching in the Process of the Further Development of the Secondary School with regard to Content). In: *Polytechnische Bildung und Erziehung*, No. 1, 1974, p. 3.

[13] Frankiewicz, H.: Polytechnischer Unterricht und Arbeitserziehung (Polytechnical Teaching and Education to Work). In: *Polytechnische Bildung und Erzeihung*, No. 2/3, 1976, pp. 69.

Education to work is an inherent component, and an important objective of mathematics teaching. But, as any objective, it is not implemented spontaneously. Rather, a careful planning of partial objectives and gradual steps is necessary for its realization, as well as hard, detailed work on the part of the teacher and the students to realize and to keep socialist norms and behaviour in the joint 'work' of the mathematics lessons.

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[14] Rubinstein, S.L. *Grundlagen der allgemeinen Psychologie* [Fundamentals of General Psychology], 6th edition, Volk und Wissen Volkseigener Gerlag, Berlin 1969, p. 740.

André Revuz

Educational Objectives for Mathematics Compatible with its Development as a Discipline

Education authorities and educational planners often wonder what kind of mathematics is needed to pursue this or that profession. It is easier to say what kind of mathematics would formerly have been needed for success in those callings than to foresee what kind will be needed in the future. Moreover it is even harder to know what the future professions of secondary schoolchildren will in fact be. On that level, then, the solution should apparently be sought in the form of 'basic mathematics required for any subsequent specialization' (or, at least, for large clusters of subsequent specializations).

But one paradoxical and frequently voiced opinion should be noted. It is that, of all possible future professions for secondary schoolchildren, one alone should be left out of the reckoning – that of a professional mathematician. Who has not heard it said of secondary or even of university-level mathematics teaching that 'The aim is not to train future mathematicians'? That remark may seem to be simple common sense, and would remain equally valid if the word 'mathematician' were replaced by 'biologist', 'agronomist', 'naval engineer', 'architect', and so on. But the rule is proclaimed only for mathematicians! Why? First of all, perhaps, because mathematics often weighs heavily on general education, which contains none of the narrower specializations.

But the questions runs deeper, and it is worth discussing, first, because the above disclaimer is so frequently expressed, and, second, because for all its apparent good sense, it tends to pull the entire teaching of mathematics in directions which seem to me to be extremely dangerous.

Let us note, in the first place, that the disclaimer is meaningful only if the following questions have first been answered (usually they are carefully avoided!):

(a) How is 'a mathematician' distinguished from 'a non-mathematician'? How should the border-line between mathematicians and others be crossed?

(b) How is 'a mathematician' trained?

(c) Is it relevant to consider and to contrast 'mathematics for mathematicians' and some paradoxical 'mathematics for non-mathematicians'?

(d) In what spirit should 'non-mathematicians' be educated in mathematics for that education to be effective?

There is widespread temptation to define 'the mathematician' very narrowly, restricting the term to research workers – or to only those (if the title is applied very strictly) who work in university mathematics departments or in institutions of similar standing whose designation varies from country to country. Even if a variation factor of 100 per cent is applied in calculating the number of mathematicians so defined, in the developed countries their total number is around one 10,000th of the working population. Certainly there is no reason to take account of so small a number in the general education of the population as a whole. But, on the one hand, has account ever been taken of it, and, if so, in what form? And, on the other, is the proportion of high-level specialists in any discipline or profession any higher?

Before pursuing the discussion, we must immediately emphasize that, in education, we have to deal with complex issues and processes stretching over a considerable number of years, and that any partial or momentary picture which is mistaken for an overall, timeless description leads to absurdities and contradictions. Let us illustrate this observation with the two questions I have just posed.

(a) Has account ever been taken of the training of future mathematicians in general education?
Taking the subjects to which research mathematicians devote their efforts, it may safely be said that virtually none of them have been introduced in those researchers' secondary or university studies, except perhaps in the very last year of university. So the answer appears to be negative. But, on the other hand, what research mathematician has not shown a very keen taste for the discipline even at secondary school? And that consideration would indicate a positive answer. However, it may, above all, be thought that the question needs refining, for the *decisive factor* in the training of the future mathematician is not *what* is learnt at the elementary levels (I include both secondary and the first four years of university), but *the way in which it is learnt*.

(b) Is the proportion of top specialists in the various disciplines and professions any greater in relation to the total working population than is the case with mathematicians?
In all cases, I think the relevant proportion is around one 10,000th. It would therefore be legitimate to apply to all

> disciplines the same rules as are applied to mathematics, namely, not to try to train future specialists. Whether in physics, chemistry, biology, geography, economics or history and so on, the same problem is to be found as in mathematics – perhaps even more acutely. There is, in fact, no question of the content of elementary education (as defined above) directly training the future researcher. And in those sciences which study reality directly, is it not, at the elementary level, far more difficult than in mathematics to present simple material which is not simplistic to the point of falsehood? Here we encounter one of the key problems of education, which, I fear, is not always clearly realized. If education is not seen as a continuous process of deepening and enriching knowledge – in which each phase is necessarily incomplete, partial and consequently provisional – instead of being led to search and invent, the pupils mind will be paralysed by rigid dogmatism and misled into worshipping false canons.

For all the change in method from discipline to discipline, the same problems are found throughout: general education cannot directly train future researchers, but, depending on the way in which it is given, it may or may not stimulate a determination to take up research.

If it is desired to train mathematicians in the narrow sense of the term, the answer probably lies primarily in the way in which education is given, and only secondarily in the choice of content. And this principle seems to be valid for all disciplines.

But should the label 'mathematician' be restricted to researchers alone, or rather – leaving aside futile arguments about titles (which none the less have their social and administrative importance) – should the work of the research mathematician be contrasted with that of someone who deals merely, as the idiom has it, with 'applied' mathematics at any level? In actual fact there is a continuous progression between the person who discovers new mathematics and the one who is content to use more or less successfully the findings and methods that others have discovered. And all of them, in varying proportions, display a mixture of different attitudes: nobody is the sole creator of all the mathematics he uses, just as, on the other hand, nobody can use any mathematics – not just efficiently, but even correctly – unless he has more or less re-invented it. And, in parallel with these two attitudes, there are two tendencies to be found in mathematics teaching, whose relationship can be schematized by contrasts: learn-understand, imitate-invent, teach mathematics as a ready-made, codified science in which there is nothing left to change, or as a science in the making, by having the pupil re-invent what is known to those who know (but not yet to him), before he himself – if he becomes a mathematician – gets round to inventing something not yet known to anyone.

Of these two tendencies, which a high-quality education should bring together, one – that of dogmatic learning by rote – is easier to apply than the other. And so all too often it becomes predominant, and then degenerates into a hand-out of stock formulae. I see several reasons for this:

(a) This tendency presents the apparent advantage of immediate efficiency. Unfortunately, that efficiency very rapidly vanishes when the method degenerates into a mere passing-on of formulae to be learnt by heart. The drawback is not imaginary: it is one which, in fact, affects all nations to some degree, but which reaches a dramatic pitch in some countries where virtually all students, on reaching university, have nothing to offer but precepts which they have learnt by heart (and have not the knowledge to apply, even in the simplest of situations).

(b) The more defective the training of the staff in both mathematics and in teaching, the more widespread the tendency becomes. A teacher whose grasp of his subject is inadequate is condemned to repetitive dogmatism and to dispensing an education in which the latter is all. To say that the pupil's understanding is not the object of this exercise is an understatement. In some far from exceptional cases, the mathematics taught is unintelligible, and if he is to understand, the pupil should begin to challenge what the teacher claims to be teaching. Badly taught pupils will become badly-trained teachers, and the resulting vicious circle will be broken only after long and painful efforts of which not even the beginning is in sight.

(c) It gives the illusion of saving time, and that what the pupils will know after any given period of schooling can be accurately pre-determined. These illusions are constantly belied by experience. But the classic explanation for failure is that the pupils do not work hard enough (true, but this is too often attributed, not to a lack of interest caused by blinkered dogmatism, but to lack of moral fibre); or that their previous education was inadequate (not realizing that the reason lies in the very same poor technique which is perpetuated in their subsequent teaching); or again, that the syllabus is too big (but there again, the size of the syllabus is not inherent, but depends on teaching methods – and of course dogmatic, imitative teaching is the very method that saturates the pupils most quickly). Furthermore, management of time in teaching activity is an extremely important and very difficult problem. Teaching time is not the same as clock-time: to stimulate a pupil's intellectual activity requires an initial lapse of time before any result is obtained, and for most teachers – who have not been required to ponder this problem long enough – that time seems too long. Yet there are periods when failure to wait destroys any possibility of intellectual activity, just as there are times when waiting would result in stagnation and the pupils must be spurred on to new questions and problems. But the real difficulty is that rules, valid for all pupils, cannot from the outset be laid down: the teacher's only reasonable course is to watch carefully the development of the pupil's intellectual activity – a technique diametrically opposed to dogmatism and the prefabricated lesson. The surprise is that, after the warming-up phase, when nothing seems to be happening, acceleration phases come about, in which the pupil's intellectual appetite is whetted by his own work, and often permits him to go far beyond the limits allowed by dogmatic teaching. Then the time-ratio is seen to be far from unfavourable to active education.

(d) Possibly the greatest 'advantage' of dogmatic teaching is that it demands no great or immediate effort of either teacher or pupil. Learning by heart requires less effort than thinking. (It may, of course, provoke thought, but it may also – probably more often – stifle it). To seek without the certainty of finding requires more effort than applying a rule backed by the teacher's authority. Dogmatic teaching draws strength from every demagogic attempt to spare pupils and citizens the salutary exertion of thinking. It doubtless gains also from the fact that the wish to provoke thought among their subjects rarely comes naturally to the authorities, and that a poorly-justified presentation of scientific truths, which it would none the less be unreasonable to reject, may serve as a corrupting model for presenting 'truths' which are less certain, but which it is desired to impose without difficulty.

Dogmatic teaching is favoured by very many pupils and students who – traumatized by the teaching of mathematics and yet needing mathematics or a mathematics diploma – see the ideal as being to 'learn' mathematics without 'doing' it. That is an intriguing and alarming paradox: the victims of mathematics teaching are actually drawn to those very techniques which are the ones least likely to improve their lot. The fact is that, in a large number of students, a fear or even a hatred of mathematics is inspired by its teaching. This prevents them trying to assimilate or to really master it. The phenomenon indicates how urgently teaching must be reformed before it can play its proper part. The first concern of a mathematics teacher should be to encourage his pupils and increase their intellectual scope. Yet the easy way out, made much easier in some countries which use mathematics teaching as a selection procedure, is to discourage and to eliminate those who are judged – in many cases, perhaps, too early – to be 'ungifted'.

Have I digressed too far from the problem of training future mathematicians? On the contrary, I am at the heart of the question. When a decision is taken on education for non-mathematicians, when it is stipulated that training future mathematicians has no place in general education, a choice is often made – sometimes implicitly, but, more often, explicitly (as when, for instance, the goal is defined to be the 'application' of mathematical concepts rather than acquiring the concepts per se) for dogmatic, imitative education, the upshot of which is almost always a slide into a mere hand-out of stock formulae, poorly understood and hence unusable. An education for future mathematicians would, conversely, be one which emphasized understanding, invention, and the presentation of mathematics as a science in the making. This ultimately is the only effective way to endow pupils with a usable instrument.

In the final analysis, would education for future mathematicians be anything other than effective mathematics teaching? And, if so, why should only future mathematicians be allowed to enjoy its benefits?

It should be noted that, as education now stands, a large number of pupils are discouraged, and that they leave school – at whatever level they may have reached – without ever having engaged in genuine mathematical activity. Moreover, those few pupils who eventually become mathematicians often do so largely in spite of, not because of the education they have received.

This division of the learners into the uncomprehending and those who reach understanding individually, while it requires more qualifying than time here permits, betrays a grave defect in the teaching of mathematics. But the situation reaches an extreme when action taken to improve it tends, on the contrary, to exacerbate and institutionalize it. Yet that is the outcome, when it is thought opportune to distinguish between education for future mathematicians and education for the others.

To train no mathematicians would be a death sentence for the discipline reducing it to fixed dogma. But it is hardly healthy either to turn mathematicians into a separate caste.

Is it a caricature to say that there is, on the one hand, a small minority of mathematicians, very much at home in their chosen field, but reluctant to emerge from it, and, on the other, a great mass of people ignorant of the mathematics they might need, and misapplying what they have supposedly learnt? The stern, at times ferocious, selection procedure, which governs admission to the mathematical caste, breeds an irrational resentment of its members among those shut out. This resentment results in the creation of that bastard form of mathematics which is all too often taught to 'non-mathematicians'. It is bastard because the concepts are often ill defined; bastard because it sometimes includes theorems which, taken literally, are false; bastard because intrinsic mathematical values, and things specific to the discipline in which they are used, remain unknown – so that in case of difficulty, the point of action cannot be found – and bastard, lastly, because an imperfect grasp of mathematics prevents the full use and mastery of that instrument. This leads yet again to dogmatic precepts in the shape of stock formulae applied with a skill slowly gained through contact with the discipline – a skill of which those who possess it are most proud, and which once more enables castes to be formed; but one which, to the extent that it is, in part, irrational, hampers the dissemination of knowledge.

To sum up the foregoing, I would say:

(a) It is impossible, and unhealthy, to draw any strict division between 'mathematicians' and 'non-mathematicians'. Where such a division is institutionalized, it is unhealthy both for the development and dissemination of mathematics and for the development of disciplines which draw on mathematics.

(b) The distinction between education for 'mathematicians' and for 'non-mathematicians' results from the general failure of mathematics teaching. Paradoxically, the defects of that teaching are used as a pretext for applying techniques which will perpetuate them.

Once that is said, only the naive would think the future to be easy for anyone who wished to improve matters. The major obstacles to be overcome include:

(a) The worldwide shortage of qualified teachers, which spares no country, but which is extreme in some so-called developing countries. The answer is a sustained process of upgrading the existing staff, not by deluging them with instructions and advice, but by involving them in collective thought and constructive criticism of the teaching function, giving it a sense of responsibility and the means of fulfilling it.

(b) The weight of unscientific traditions, and the falling back on reassuring dogmatism. The difficulties of accepting the relativity of knowledge and scientific truth, and of adopting an attitude halfway between blind faith and total scepticism hamper the spread of culture among new population strata. The individual exercise of intellectual freedom is not always a spontaneous process, nor is it always encouraged by the authorities on whom the organization of education depends.

(c) The desire for immediate profitability, which tends to foster the creation of small hyper-selective and hyper-specialized groups and of a great mass of people trained for mindless obedience. It is superfluous to emphasize the threat this poses to a healthy democracy.

What, then, should be done?

It must first be clearly understood that this is a very long-term task, and distant objectives must not be overlooked when applying short-term measures. These cannot in themselves lead to the ideal objectives, but one must at least be sure that they are leading in the right direction rather than – as is, I think, too often the case – helping to entrench existing defects.

A dynamic attitude must be adopted towards progressive change, summing up the initiative and responsibility of all who work, at whatsoever the level, in education systems.

As regards the teaching of mathematics itself, its style must be the prime consideration, and its content only secondary – or rather, it must be realized that style and content are inseparable and that what is called the same content can, in reality, be very different when the teaching style is changed. A productive teaching style is one informed by the spirit of research. It is unhealthy to present even the most elementary of theoretical outlines without first having posed some of the problems that the theory sets out to solve. Moreover, to think it is easy to pose problems is naive. Very often, it is the finding and correctly formulating the problem that has been one of the most awkward stages in the development of science. So, education needs time. Time which, to most teachers and their pupils, will seem to be wasted time.

Time must also be devoted to the search for solutions, and to leaving the pupils to tackle the problem they are to solve. That is an essential part of teaching. And it needs the ability to choose a problem which needs thought, but which it is not beyond the pupils' ability to solve. Another essential is the ability to wait. A mind cannot be forced to create and to invent any more than a plant can be made to grow by pulling it. All that can be done is to provide favourable conditions for the flowering of creativity. It is a matter, therefore, of genuinely motivating the pupil and patiently awaiting his reactions, and discussing his efforts with him with a benevolence untainted by condescension. At this stage, the teacher must stay in the background. He must encourage the pupil to provide an effort, rather than attempt to suppress it. In such a situation, the pupil often has good ideas, but cannot, himself, see where they lead. So he has no spontaneous desire to take them to their conclusion. Here the teacher can intervene – not to give him the answer, but to urge him to persevere. On the other hand, when the pupil has found the solution, the teacher must use his full authority to demand that the argument and the reasoning be clearly and thoroughly set out. Obviously such a procedure requires staff who are highly trained in science, in teaching and in psychology. It requires small classes. Equally obviously, these requirements are, in the short term, unattainable on a large scale in a large number of countries.

What then can be done here and now? This may be where a valid contribution can be made by the training of future mathematicians – a term used this time in its widest possible sense.

Within the overall teaching of mathematics, islands could be created in which teachers attempt to apply the above principles. But they would be viewed as models to be introduced gradually throughout the education system, rather than as seminaries in which an élite is separated off from the mass. I do not underestimate the risk of the latter possibility prevailing, but I see no alternative to running that risk, while attempting to avoid it. I would also hope that an élite trained by open methods will be less inclined to close in upon itself and defend its privileges than is an élite recruited by an education dominated by dogmatism and out-and-out selectivity.

The content of education should be based on these two principles:

(a) It must be suited to training both future mathematicians and future scientists who need mathematics.

(b) Mathematics should not be treated as a discipline apart, but in conjunction with the other disciplines.

At the end of secondary school or university entrance level, the content might well include:

(a) The basics of calculus: differential and integral, both theoretical and applied.

The intention is not an aimless calculation of integrals, but an understanding of how certain phenomena may be approached through a differential equation, and a study of the implication of its solutions. In a word, one must not fall into the very widespread trap of teaching mathematical techniques (differentiation, integration) without ensuring that the fundamental ideas brought into play by those techniques are understood, or without having applied those techniques to specific and carefully studied examples.

(b) Elements of algebra, and especially of linear algebra.

Geometry, a privileged field in which transformation groups come into play, should provide motivation for and permit the application of most algebraic concepts at this level. There will, of course, be an outcry against a sacreligious algebrization of geometry, and a rejection of the 'geometrization' of algebra. But a study of geometry is useful for future mathematicians, and for users of mathematics, so long as it is not made into a separate, self-contained branch. Rather, it should be treated in close conjunction with the other branches of mathematics.

(c) Elements of probability theory, which would give insights into the 'probabilist' spirit and provide skill in the use of statistics (in their right place and applied honestly!).

This subject should be linked to analysis, for which it could supply motivation and applications.

Here again, however, content must not become an obsession. The question whether a little more or a little less geometry should be included seems to me laughable in comparison with the fundamental issue – how is the content to be taught?

It must also be realized that teaching styles need to change, not only at the level for which I have thought it apposite to outline the content I believe to be desirable. They must also change in the early years of secondary school, and in the primary school. Here, the enormous numbers of teachers and pupils involved make the task of changing styles extremely difficult. They provide another reason for restricting immediate action to the creation of 'experimental islands', with a view to their subsequent gradual extension. Nor must one be blind to the fact that the inherent educational difficulties are exacerbated by social and political obstacles even more formidable than the former. For if those 'islands' are successful, they will create inequalities. These may be rejected from the outset because of strictly egalitarian concepts. In that case, progress would be blocked throughout education. The beneficiaries of the 'islands' would have to realize that their good fortune – even if merited – gives them responsibilities rather than rights towards others.

In conclusion, my thesis may be summarized as follows:

(a) The training of mathematicians is indispensable. Optimally, they should be able not only to cultivate mathematics per se, to marshal the whole range of mathematical instruments, to sharpen them and to create new ones, they should also be intellectually and psychologically open to dialogue and collaboration with specialists in other disciplines.

(b) Only at a very advanced level should mathematicians start to specialize. Their education prior to that point should have painstakingly fostered an intellectual atmosphere conducive to a deep understanding and inventiveness. In a word, it should have been informed by a spirit of research. There is no reason for restricting that type of education to future mathematicians only. In the final analysis it is the only efficient mathematics education.

(c) It is illusory to imagine that this kind of education could be generally introduced in any country in the immediate future. One of the major obstacles is the shortage of qualified staff.

The temporary solutions are, on the one hand, 'islands of excellence', intended as models for subsequent gradual extension, and, on the other, specialized education of mediocre cultural value to enable non-mathematicians to acquire a minimum of mathematical ability, since time and resources permit no more. According to the spirit in which they are used, these may be the start of a real improvement, or – on the contrary – they may consolidate and perpetuate the worst defects of the teaching of mathematics.

It is to be feared, in view of the immensity of the task, that neither the goodwill nor the necessary energy will be available to bring about a high-quality mathematics education everywhere.

The solutions I propose may be judged unrealistic. In my view, however, the height of unreality is to think them unnecessary.

Frederic van der Blij, Sven Hilding and A.I. Weinzweig.

New Goals for Old: An Analysis of Reactions to Recent Reforms in Several Countries[1]

The last two decades mark a period of unprecedented change in the mathematics curricula of most countries of the world. So radical have been these changes that they are often referred to as a revolution! The blanket term 'new mathematics' has come to be applied to all varieties of the new programmes introduced into school systems during this period, programmes which differ considerably from one country to another, and often, even, from one another within the same country. What the term has meant, however, in all school systems is a restructuring of the subject, with the inclusion of new topics and the deletion of some traditionally taught material, but all viewed from a new perspective. These changes have had far reaching implications for the training and retraining of teachers, and for the allocation of resources to effect these changes.

Within a relatively short period of time, the tidal wave of 'new mathematics' has spread over the world, engulfing school communities and sweeping away both items of value as well as curricular rubbish. In some places, the waters are just beginning to rise (e.g. Hungary); in others, the water is still rising (e.g. Indonesia, Nigeria); in some, it has reached its peak (e.g. France, Belgium); in yet others, the wave is receding (Scotland, Sweden).

Why did these changes take place? Who provided the impetus for them? How were they implemented? How were the changes spread? What were the nature and effect of the changes?

To these questions we now address ourselves.

The Reasons for Change

In order to understand what prompted the feeling of a need for change, one must first understand the climate in which it germinated. The period which followed World War II was one of intense mathematical activity – a veritable explosion in mathematical knowledge. New results were being announced at a fast and furious pace. New methods and techniques were being developed. New mathematical concepts were being created. It was a heady, exciting period. The general direction of these innovations was towards greater abstraction of thought. This, in turn, called for a much more careful formulation of ideas and a greater precision of language. Sets, and, to a lesser extent, functions, emerged as unifying concepts, exemplified in the Bourbaki[2] undertaking.

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[1] Paper presented at conference "Stability & Change in Mathematics Curricula, 1960–1980" (Institute of Mathematical Didactics, University of Bielefeld, Federal Republic of Germany, 7–12 Jan 1980).

The decisive role of technology in the war led to a great emphasis, in the post-war period, on technology as a force for progress and development. This, in turn, created a need for more technologists of various kinds. This was true both in the developed countries and (even more so) in the many newly independent developing countries, which looked to technology as the key to their development. Mathematics, which had played so important a part in these impressive technological achievements, was given special emphasis in the drive for more and better-trained technologists.

The student populations in most countries were greatly expanded, particularly at the secondary level. The move from an 'élitist' to a 'comprehensive' school system was one reason for the expansion in many developed countries. Even in countries like the United States of America, which had always provided 'comprehensive' secondary schools, the student population expanded. A greater proportion of students remained in school after attaining the minimum legal age for leaving so as to qualify for higher education, and, later, to take advantage of the many new career opportunities which the post-war world had opened up.

In the developing countries, more universal school systems were being created, and opportunities for education in universities increased dramatically.

All of these factors combined to create a climate conducive to the acceptance of change.

Not surprisingly, changes were first seen at the university level, with the introduction of new courses based on the newly emerging mathematics. These courses were presented with a customary concern for rigour, and it was not long before the university mathematicians were voicing their growing dissatisfaction with the preparation of incoming students. In the United States, this led to the establishment, in 1951, under the direction of Prof. Max Beberman, of the University of Illinois Committee on School Mathematics (UICSM) at Urbana,

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[2] The name adopted by a group of French mathematicians who set out to re-structure mathematics on a set/axiomatic basis.

Illinois (at present the base of the Curriculum group of the International Association for the Evaluation of Educational Achievement (IEA) Second International Mathematics Study). The objective of UICSM was to improve the mathematics courses on offer in the secondary schools.

There was also a desire among many mathematicians to infuse the secondary school curriculum with the excitement of the new developments in mathematics, and to bring school courses more into line with university trends. This was particularly true in France, where the mathematicians who were at the heart of these new developments exerted a considerable influence not only nationally, but internationally.

Political leaders in many countries in the western world, in Japan, and in many developing countries were concerned about the need for a broad education in technology to meet the rapid growth of new fields of industry. They believed the curriculum, then current, to be too outmoded to cope with the impending technological era, and pressed for change. One manifestation of this concern was the OECD's sponsoring of a series of International Conferences (Royaumont 1959, Dubrovnic 1960, Paris 1961). These produced the famous *Synopses* – Synopses for secondary school mathematics and mathematics for physicists and engineers.

The expansion of the secondary school population flowing from the wider opportunities for secondary education led many university mathematicians and secondary teachers to question the suitability for the broad masses being enrolled in the schools of a mathematics curriculum originally designed for an intellectual élite. Secondary school teachers in many places (England, Scotland, Sweden) were, at once, eager to modernize the curriculum, anxious to find new approaches to meet the needs of less gifted students, and excited and stimulated by the prospect of teaching new topics like probability and statistics.

Finally the 'band-waggon syndrome': once the movement for change in a mathematics curriculum gets under way in certain 'culturally leading' countries, there naturally develops in other countries an anxiety not to be 'left behind' in what comes to be thought of as a general 'revolution'. This, too, created a powerful pressure for curriculum reform.

The Impetus for change

The winds of change were blowing in many places. These infected different groups of individuals – university mathematicians, teachers and government officials. They took different directions in different places. We look at three cases, those of England and Wales,[3] the United States and France.

In England, the impetus for reform began with the Jeffrey Report (1944). This proposed changes in the syllabus of the examinations taken at the age of 16. It called for a 'fusion of mathematical subjects' to replace the traditional separation of the subject into the compartments of geometry, algebra and arithmetic. Examination papers would be made up of a mixture of problems. Candidates would have complete freedom in their choice of a method of solution. Calculus would be included as an option. The expressed aim was 'to bring mathematics more closely into relation with the life and experience of the pupil'. Applications of mathematics would be included. The time for these would be found by a 'reduction in the amount of formal geometry' and the 'exclusion . . . of heavy calculations and manipulations . . . emphasis [was to be on] the all-important areas concerned with functionality'. The report dealt with secondary education, and it focused primarily upon content. Nevertheless, it is interesting to note the call for a unified curriculum, and the emphasis on those applications which relate mathematics to the experience of the pupil.

The Jeffrey Report, published by the Mathematical Association, set the direction for reform at the secondary level. The Mathematical Association also published in 1956 a report on Primary Education. This report dominated opinion on primary mathematics for the next two decades. It took the position that children developed 'at their own individual rates', and that they 'learn through their active response to the experiences that come to them'. 'The aim of primary teaching', it said 'is the laying of the foundations of mathematical thinking about the numerical and spatial aspects of the objects and activities which children of this age encounter'. The focus here has shifted from content to the child, but the emphasis, again, is on relating the mathematics to the experience of the child – indeed on drawing the mathematics out of these experiences! What is amazing about this forward-looking report is that it was prepared primarily by university mathematicians. Unfortunately, they had no effective links with teachers in the schools (neither primary nor secondary), and so could do little.

More effective links were forged by the newly-formed Association of Teachers of Mathematics (ATM). This came into being through the common desire of a group of teachers to investigate the place of structural materials, manipulative devices and apparatus, in the teaching of mathematics. ATM emphasized teaching methods rather than content. Its members were learner-centred. Their journal, *Mathematics*

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[3] In the United Kingdom, there are three different systems of education: those of England and Wales, Scotland and Northern Ireland. In what follows, 'England' will be used where, strictly, 'England and Wales' should be used.

Teaching, their *Some Lessons in Mathematics* (published in 1964) and their *Notes on Mathematics in Primary Schools* (published in 1967) exerted a considerable influence on teachers.

The call for change was made by university mathematicians, but the impetus for change came from teachers. In 1961-62, several 'projects' were launched by independent groups of teachers. They included the School Mathematics Project (SMP), the Midlands Mathematical Experiment (MME) and the Contemporary School Mathematics Project (CSM). These projects were designed primarily for the academically-oriented secondary school students. They were entirely unsupported, financially or otherwise, by the Ministry of Education. Writing was done by practising school teachers. They held to the English tradition of 'applied mathematics', and avoided the excess of abstraction which characterized some of the 'new math' curricula developed in the United States and on the European continent. The material, in short, was teachable to real students.

At the primary level, one of the movers was Edith Biggs. In an early Schools Council bulletin, *Mathematics in Primary Schools* (1965), her proposals reflected the influence of Piaget's theories about the formation of concepts, and of Dienes' ideas of structured activity and multiple embodiments. However, her recommendations were based on classroom experiences – on what had been found to work rather than on any theoretical considerations. The Nuffield Mathematics Project was initiated in 1964 in order to implement some of these ideas. The goal was 'to produce a contemporary course . . . . designed to help children connect together many aspects of the world around them, to introduce them gradually to the processes of abstract thinking, and to foster in them a critical, logical but also a creative mind.'

Thus in England, the impetus for reform came from the teachers, with the focus of effort on teaching and child-centred learning, and a heavy emphasis on the use of the environment as the source of mathematical experiences, and on the applications of mathematics to the solution of problems arising in the real world.

Pressures for change came from a different direction in France. French mathematicians, especially the members of the Bourbaki group, played a very strong role in the explosion of mathematics during the time of World War II, and exerted a tremendous influence internationally on research in mathematics. It was only natural that they sought to imbue the school curriculum with this new spirit. The concern was primarily with content, reflecting what has come to be called the Bourbaki approach – formal, abstract, rigorous. The emphasis was on precision of definition and on the careful use of language.

In the United States, the winds of change blew from several directions. UICSM, mentioned earlier, focused on the content of secondary school courses, emphasizing algebraic laws, deductive principles, relations and functions (perceived as sets of ordered pairs), and 'solution sets' for equations and inequalities. There was also an attempt to introduce, at the secondary school level, the same rigour and precision of language as characterized much of the new mathematics then being done on the frontiers of research. Some thought was given to teaching methods, which Beberman described as 'discovery method pedagogy', but this was much less evident in the later books on deductive method and vector geometry. The middle school programme (ages 11–13), with fractions appearing as operators ('stretchers' and 'shrinkers') and motion geometry, was more learner-directed and less formal.

At the elementary level, the University of Illinois Arithmetic Project (UIAP) (1958), directed by David Page, introduced boxes as place-holders, and used them to deal with solutions of equations and inequalities as well as with functions and the 'laws of arithmetic'. Content, again, was the main concern with some emphasis on how to teach it.

The Madison Project (1957), under the direction of Robert Davis, 'was created in order to revitalize the teacher education program, . . . in order to achieve a closer relation between the studies of prospective teachers and the operational problems of high quality schools'. The product was supplementary materials for elementary schools. With the needs of the learner chiefly in mind, they laid emphasis on the 'discovery approach', and gave prominence to manipulative materials.

The Minnemast Project (1961), under Paul Rosenbloom, set out to produce an integrated mathematics-science curriculum for five to eleven year olds. The course was highly orientated towards the child's environment, with an emphasis upon concrete experiences, from which mathematical insights and ideas should develop.

Thus, the first phase of curriculum reform in the United States took two different directions. On the one hand there was a stress on new content, with a concern for greater precision in mathematical language and ideas. This approach characterized the UICSM secondary programme in particular and the UIAP elementary programme to some extent. On the other hand there were the child-centred, activity-orientated Madison and Minnemast projects. In either case the reformers were mostly university people.

The Commission on Mathematics, appointed by the College Entrance Board in 1955, produced its 'Programme for College Preparatory

Mathematics' in 1959. This cited, as an argument for reform, the spectacular growth of pure mathematics, and the extensive development of new concepts and methods, accompanied by the successful application of both new and classical mathematical ideas to disciplines which previously had eschewed the use of mathematics. The Commission argued that secondary school mathematics should present new content so as to bring students more efficiently to the frontiers of pure and applied mathematics, in order to meet the growing need for mathematically-sophisticated scientific manpower. The Commission suggested the inclusion of topics from logic, modern algebra, probability, and statistics, and they recommended that plane and solid geometry should be integrated in a single course; that trigonometry should be merged with advanced algebra; and that inequalities should be studied as well as equations. The entire curriculum would be unified through the judicious use of the deductive method, the process of searching for patterns, and the introduction of such structural concepts as sets, relations, and functions. Thus, the report not only argued for change, but indicated in some detail the direction to be taken.

The launching of Sputnik I by the Union of Soviet Socialist Republics created a shock wave throughout the United States, which, among other effects, shook loose large sums of Federal funds for educational innovation. These spawned a number of new curriculum development projects, and many conferences. Among the former was the School Mathematics Study Group (SMSG). This was organized by mathematicians in 1958. Its objective was to improve secondary education by preparing 'sample texts'.

Following the Woods Hole Conference, Jerome Bruner put forth his dictum: 'any subject can be taught effectively in some intellectually honest form to any child at any stage of development'. This stimulated the infusion into the elementary school curriculum of new topics previously taught only at more advanced levels. Thus, SMSG, on moving into the development of an elementary programme, included for children of primary age informal geometry, probability, algebra and number theory. The focus was on content and the emphasis on structure. 'Sets' made their appearance as a unifying concept. Words and phrases like 'the number property of a set,' 'commutative', 'associative' found their way into the jargon of elementary school mathematics. While the writing teams included teachers as well as professional mathematicians, their output was, in fact, dominated by the latter.

The school systems in England and in the United States are very decentralized. The local authorities are all free to devise their own curricula, and, in England, this freedom is delegated to the individual school. The advocates of reform were generally enthusiastic zealots with a grand vision of the future. They travelled a great deal, with a mission to push the 'new math' at various meetings and conferences. Even in countries with centralizied systems, not only had the ministry to be convinced of the value of the reform, but, just as in the United States and in England, the teachers had to be convinced also, to say nothing of parents! Inevitably, the tendency was to overstate the case and to promise salvation.

The enthusiam spread like wild fire. While it did not consume everyone, enough teachers and others were converted to silence the agnostics and the atheists. The prophets of change travelled on to other countries, and there, too, they engendered excitment, not only among the teachers, but among parents and the media. Few decision makers would risk the charge of being 'left behind' in a movement so widely heralded.

Implementing the Reform

The realization in practice of so radical a reform in the mathematics curriculum could only be achieved with the active participation of the teachers. Thus, it was crucial to success to convince teachers of the need and the value of the changes advocated, and to help them to implement the new ideas. School systems tend to be very conservative. Parents expect their children to be taught 'good' mathematics. This, for most of them, is the mathematics they were taught in school. The society also has its own expectations of what is required. But even where there is a recognition of the need for new knowledge, there is usually an unwillingness to make room, least of all by shedding good 'basic' content. Finally, it is a truism that experienced teachers are more comfortable with what is familiar to them. Hence they tend to form a core of opposition to major change. When change is urged from above, – by ministries or by the university people, (as for example in Sweden, France and the United States), then teacher resistance must be overcome. So it became evident that to effect so revolutionary a change would require a large-scale teacher re-education programme as well as a renewal of the teacher preparation programmes.

During the sixties, the Government of the United States financed large-scale in-service training programmes for secondary school teachers. In 1969, the French government established a number of institutes for research into the teaching of mathematics (IREM), which were required not only to undertake research into mathematics education, but were also to provide training for teachers in service, and to prepare new teachers. Sweden adopted another approach. Teachers, both primary and secondary, were encouraged and coerced (in a nice combination!) to devote a great deal of time to the study of the new content in mathematics, in order to be able to teach it. The British set up 'teachers' centres' to encourage, reassure and assist the teachers in implementing the new curricula.

Those who sought change struggled with those teachers who resisted change. The in-service programme in the United States attracted only a small percentage of the secondary school teachers, and none of primary level. Even in England, while the teachers' centres achieved a great deal, there remained a large proportion of teachers who took no advantage of the opportunities on offer.

In certain countries, like France, the Federal Republic of Germany, and even, to some extent, the United States, secondary school teachers are specialists in mathematics, and are accordingly better prepared to handle the new curriculum, especially those of the teachers who are younger and more recently graduated. They tend to identify more with mathematicians, and are hence prone to support the reforms advocated and pushed by the university mathematicians. On the other hand, for the elementary school teachers, such in-service provision as there was could scarcely compensate for their own impoverished mathematical background.

In Sweden, however, where secondary teachers teach one or two subjects other than mathematics, the profession was more antagonistic to change. The result was that the new programmes in Sweden had to be modified considerably. Set theory, references to the commutative and associative laws have been eliminated, and the study of probability has been reduced. In the United States, a counter current set in at the elementary level. This was the 'back to basics' movement, which has its manifestations in other countries as well. Thus curriculum reform seems to have 'taken' somewhat better at the secondary level than at the primary.

The Changes in Content

The most important change was the introduction, in many countries, of an integrated curriculum at the secondary level. The need to introduce new branches of mathematics such as probability, statistics, computer science, etc. raised the problem of whether these should be taught as separate courses together with the traditionally separate courses in algebra, geometry and trigonometry, or whether to attempt to integrate the new branches into the old. Instead, influenced by the spirit of 'modern' mathematics and the employment of set theory, developers set out to create a unified curriculum for mathematics with integrated text-books and examinations.

Some called for the banishment of traditional Euclidean geometry, advocating instead, the study of geometry through transformations or the treatment of geometry through linear algebra.

In many countries, the secondary curriculum became much more 'rigorous', with an emphasis on the precise formulation of mathematical notions and on the structure as a deductive system. In the United States, the most common approach to geometry was to underpin the deficiencies in Euclid's axioms by using the Berkhoff-Beatty axioms. In France, the approach was to use a linear algebra. The Federal Republic of Germany seemed to favour an axiomatic formulation of 'transformation' geometry.

The formal, rigorous approach of much of the 'new' mathematics was manifested not only in geometry, but in everything, including the calculus. This, too, tended to be quite abstract, with a deductive development being the prime concern. This trend was pure mathematics in its 'purest' form. Some of the formalism which had penetrated secondary mathematics found its way into the primary curriculum as well.

In other countries, the desire to meet the needs of the many, generally the less academically able students finding their way into secondary schools as a result of the democratization of the school system (as well as the desire to retain national traditions), led them to follow a different course. In these countries, mathematics was treated much less formally, with a great emphasis upon its relevance to the environment. England is an excellent example. There, the focus was on activities from which were drawn the mathematics problems. Applications, too, played an important role.

Probability, statistics, logic and computer science are now a part of the secondary school curriculum, and some of these topics are even found in the elementary curriculum. The teaching of the calculus is now much more widespread, but solid geometry, and the 'solving' of triangles in trigonometry have virtually been eliminated. Geometry has assumed a more important position in the elementary curriculum.

The new curricula have generally de-emphasised what had been a major feature of the traditional curriculum – computation. The argument for reducing the amount of computation traditionally taught was cogently put in the Report of the Cambridge Conference *Goals for School Mathematics*[4] (1963). This stated: 'Long pages of addition and multiplication problems add nothing to a student's understanding of the processes involved, nor do they teach him when to add or to multiply'. The Conference did, however, agree 'that reasonable proficiency in arithmetical computation and algebraic manipulation is essential'. But it was affirmed that 'entirely adequate practice in computation can be built into problems that . . . genuinely attract the student's interest'.

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[4] Published for Educational Services Inc. by Houghton-Mifflin Co. (Boston '63).

In practice, unfortunately, quite often the first part is followed, but the second part overlooked. Many critics of the new reforms have pointed out that the drive towards more rigour, and the emphasis upon structure have produced students who can 'manipulate symbols', but lack mathematical insight. These critics, who are to be found both in mathematics and in other disciplines where mathematics is used, complain that students have learned only to reproduce some definitions, and to adopt a new terminology. They lack the ability to use them in solving problems. Some critics have even contended that the very spirit of mathematics has been hidden behind a cloud of language, definitions and axioms – that real mathematics has disappeared!

The role of the external examinations

External examinations and tests have had two major effects on the introduction of new mathematics curricula. The first concerns the rate at which curricula have been adopted. The second concerns the way in which curricula have been interpreted and implemented.

In almost all countries, the new mathematics movement initially affected only the relatively few classes and schools involved in small pilot projects. This stage was followed by a steady growth in the number of classes taking part, as trial extended upwards through schools. In school systems where public examinations were taken by students, the problem of mismatch arose as between the new curricula and the traditional. This was usually overcome by providing 'alternative papers', by which achievement in the new curriculum was examined. Thereafter, 'traditional' and 'modern' papers were offered in parallel for a number of years, with schools exercising the choice of which paper individual students were entered for. Eventually, the 'traditional' option was terminated in most countries. The length of the transitional period had a direct relationship to the rate of adoption of the new curricula, and hence a direct relationship to the stresses set up within the educational system. In countries where the period was relatively short, the pressure for teachers to adopt the new curriculum was great, and the associated demands for in-service training and the provision of resources such as textbooks, teacher manuals, etc. was correspondingly great. These demands were seldom to be adequately met. In these countries, also, teachers who were opposed to the new curricula were forced into making the change. In many cases they did so unwillingly and without training. Many were therefore unable to adopt the new approaches to mathematics teaching which were necessary for the full potential of the changes to be realized. On the other hand, where the period in which alternative papers were available was extended indefinitely, as is still the case in England, the rate of adoption slowed considerably since the stimulus for change, which many teachers need, was not there.

In all school systems, external examinations or testing programmes exert a significant backwash effect upon what teachers teach. And it is in the nature of examinations and tests that particular items can distort the teaching of particular topics. For example, questions on set notation and on Venn diagrams have often led to 'sets' being taught as an isolated topic when it was intended that the idea of a set should be used throughout mathematics as a unifying concept. Similarly, numbers to bases other than ten, and modular arithmetic, which were originally included in new curricula in order to clarify the concept of a 'base' for counting, and to generalize notions of number systems, have often come to assume a prominence in teaching which is out of proportion with their importance. Tests and examinations are among the influences which have caused many teachers to overemphasize the pedantic use of new terminology at the expense of the ideas behind the terminology.

It would, however, be misleading to suggest that the influence of tests and examinations has always been negative. Well-constructed items can convey the flavour and the intention of the new curricula. And well-balanced papers can indicate the relative importance of the various new topics. It is also evident that the necessity, even for unwilling or conservative teachers, to match the curriculum taught to what examinations expect, *increases their application*, if not their willingness, when it comes to trying to teach the new material. In spite of these positive possibilities, it must be admitted that the goals are not always reached. It is not easy to avoid the natural tendency to choose those test items which examine only skills and behaviours that are easiest to measure.

Concurrently with the introduction of new curricula in nearly all countries there has been a dramatic rise in the proportion of students remaining at school to take public examinations. In some countries the proportion of students electing to study mathematics has also risen. Examination syllabuses have not always been adapted rapidly enough to allow for the wider range of abilities of the students they cater for. In this situation, significant groups of students have been presented with courses of study designed for more able students. Such courses are more abstract and academic than would produce optimal learning. In this sense, examinations have obstructed moves to adapt the programme to the needs and abilities of less able students. This, of course, has contributed to the doubts some teachers have had about the value of the new programme, even if such doubts were not a result, *per se*, of the new mathematics.

Residual effects

While it is too early to predict the lasting effects of the upheaval in mathematics education of the last two decades, some of the beneficial outcomes associated with the developments can be identified.

In all countries, there has emerged a cadre of influential mathematics educators with perspectives and insights into mathematics, into its teaching and learning. In many countries, the machinery for, and the habit of, co-operation between teachers of mathematics in secondary schools and in higher education with learning theoreticians has been established for, at any rate, the process of curriculum development in mathematics.

Within the subject, at secondary level, a way to teach mathematics as an integrated subject through such unifying concepts as functions, sets, etc., has been found. This seems likely to remain a permanent characteristic of secondary mathematics curricula for most countries, with more attention being paid to underlying structures and principles than was customary in the past. Many countries report a desire to bring into effect 'continuous' development of curricula, i.e. to make minor changes when the need is felt rather than to make major changes after lengthy periods of curricular inactivity. Thus, currently, some countries are adjusting their curricula by changes in emphasis to overcome deficiences revealed after a few years' experience with the new programme. Topics which have not fulfilled expectations are being modified, and the need for regular practice in some skills is being re-emphasized. It is abundantly clear that just as drill in skills without understanding is educationally unprofitable, so is understanding without the regular maintenance of skills.

The ferment in mathematics education has almost certainly led mathematics teachers to concentrate more upon educational outcomes, and to consider the relationships between what they are teaching, how they are teaching it, and what the student is learning. The updating of the content of mathematics curricula was overdue. While the debate on what content is necessary and desirable will continue, it is certain that the curriculum will be subjected to more regular and critical scrutiny in the future than it was in the past.

At the international level, as a result of the various projects, conferences, debates and consultations, there has come into being an active network of mathematics educators, who share a common interest in the school mathematics curriculum and in teaching methods.

Compare, for example, the International Congress of Mathematicians, in 1954 in Amsterdam, in 1958 in Edinburgh, in 1962 in Stockholm, where the education section was small, or even trivial, with the success and huge attendance at the meetings of ICMI in Lyon, Exeter and Karlsruhe in the seventies. The cross-fertilization of ideas within this community of mathematicians and mathematics educators will certainly continue to be of great benefit to all the children in the world.

Alan Osborne

The NCTM PRISM[1] Project: An Attempt to Make Curriculum Change More Rational and Systematic[2]

Designing and implementing curriculum change is an expensive undertaking. Some of the resources needed for investment in curricular change are easy to identify. Time and money are examples of such tangible resources. Other resources are of a more ineffable character and are possibly more significant if the change is dramatic and significant. For example, it is difficult to plan and budget for creative energy and commitment.

Resources for education are limited, and mathematics education is no exception. The maintenance of current, ongoing programmes absorbs resources. The planning, design and implementation of curricular change call for investment over and above that required for maintenance. This is because change involves the development and production of materials, the evaluation of their revisions and the in-service re-education of teachers to ensure their acceptance of the change and their adequate understanding of the materials and of the associated teaching methods. These are but a few of the processes associated with change that consume additional resources. Before making such an investment, wisdom would dictate that the risks and the expectations associated with such a venture should be analysed.

One element of risk that has seldom been examined in planning for change in curricular practice is the individual nature of the variations in the preferences and priorities for change found among those who have a stake in mathematics education. When a curricular alternative is developed, it is seldom that such groups as teachers and parents will have been asked for their opinions on its value and worth. In most cases, the modifications of curricular practice in mathematics will have been generated by a small set of leaders in mathematics and in mathematics education. Such individuals will have been guided by their own judgement of what is important in mathematics, by their own perceptions of the needs of society, and by their own feelings for the deficiencies in the performance of the school mathematics programme. Frequently, the modifications have included distinctive and innovative methods of teaching and approaches to content. Although many of the plans for modifying the curriculum have been well-conceived, they have failed to be successful. Their impact, and such effect as they have had, have been limited to the initial years of their conception and implementation. To this extent, they represent a waste of resources.

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[1] Full title: National Council of Teachers of Mathematic's Priorities in School Mathematics Project.

[2] The material in this article is based upon work supported by the National Science Foundation, an agency of the federal government of the United States of America, under Grant Number SED77-18564. Any opinions, findings, and conclusions expressed in this account are those of the author. They do not necessarily reflect the views of the National Science Foundation.

If better information were available on the acceptability of alternative curricular practices to concerned groups, the developers of curricular alternatives could either design more acceptable alternatives or they could plan processes of implementation that would gain support. The PRISM Project is an attempt to gather such information in the United States of America. It is a part of a larger project of the National Council of Teachers of Mathematics (NCTM). NCTM is the largest professional group in the United States uniquely concerned with the teaching and learning of mathematics in the elementary and secondary schools. NCTM, through the action of its Board of Directors in 1976, adopted as its top-priority and five-year goal the development of specific curriculum recommendations suitable for the decade of the 1980s. The recommendations were formulated independently of the PRISM Project. But those responsible were cognizant of some of the findings of the PRISM information gathering. And the planning of the activities and the policies inspired by NCTM to support the recommendations was based, in part, on the PRISM findings. NCTM has been concerned in that previous attempts at curricular change have seldom had any lasting effect. They have typically failed to consolidate productive practices. And they have had a propensity for ignoring the findings of research. It is hoped that the information gained through PRISM can alleviate these problems by enabling change to take place in a more systematic and rational manner.

The sections that follow describe the PRISM Project and the design of the instruments used to gather information. They also discuss some of the findings in terms of their potential for making an impact on curricular change. More extensive reports of the findings of the PRISM Project will become available in publications of NCTM due to issue in the autumn of 1980.

Design and Instrumentation

The PRISM Project was designed to collect information about curricular practices in mathematics desired by teachers and other interested parties. These could then be compared with actual practices in the schools. Two surveys were made, one to determine 'curricular preferences', the other to elicit 'curricular priorities'. 'Curriculum' was broadly defined to encompass mathematical content, tools and modes of instruction, and levels of student need. The project began in the autumn of 1977. It has only recently been completed.

The instruments used for the first-round or the survey of *'curricular preferences'* were designed to enable respondents to make a choice of curricular alternatives. The curricular alternatives were presented to respondents both in relation to nine broad areas or 'strands' of mathematics and according to six 'categories' of curriculum planning. The 'categories' considered were content, goals, resources to be used in giving instruction, teaching methods, who should learn the content and when, and the use of calculators. The latter topic was included because it is a particularly critical issue for many people. Each of the 'categories' had items that reflected issues or problems of current significance. Each 'category' was taken as a base for the construction of a cluster of items in relation to each of the nine 'strands' of mathematics. The nine 'strands' were: FD–Fractions/Decimals, AL–Algebra, WN–Whole Numbers, GM–Geometry, PS–Probability and Statistics, RP–Ratio/Proportion/Percent, PB–Problem Solving, MS–Measurement, and CL–Computer Literacy. The structure of the 'strand-category' matrix is given below:

Categories of Items

Strands	CONTENT	GOALS	RESOURCES	METHODS	WHO/TIME	CALCULATOR
FD						
AL						
WN						
GM						
PS						
RP						
PB						
MS						
CL						

Strand-Category Matrix

Clusters of items were created for the cells in the strand-category matrix. Each cluster exhibited alternatives in curricular practices. An example of a cluster of items is given below. It is selected from the geometry strand, and it belongs to the category of *goals* in teaching geometry.

STEM

Imagine that there are available several sets of instructional materials for geometry. The materials differ in that each emphasizes one of the curricular goals listed below even at the expense of other goals. Suppose that the materials are equivalent in terms of other factors. Indicate the degree to which emphasis on the stated goal should be a positive influence on the decision to use or to buy these curricular materials.

a. Strong positive influence
b. Somewhat positive influence
c. No influence or undecided
d. Somewhat negative influence
e. Strong negative influence

Geometry is taught:

ITEM

21. To motivate students who dislike computation.
22. To develop job-oriented skills.
23. To appreciate historical and cultural development.
24. To learn to make proofs.
25. To develop spatial intuitions about the real world.
26. To learn to read and interpret mathematical arguments.
27. To practice arithmetic and algebraic skills.
28. To develop logical thinking abilities.
29. To develop skills and knowledge needed by the consumer.

Two teams of item writers, representing a wide variety of backgrounds in mathematics and in mathematics education, prepared the clusters of items for the cells in the strand-category matrix. To inspire them, the staff of the project had prepared a background paper for each of the strands. These were designed to stimulate the item writers to consider alternatives, issues and problems associated with each strand. The initial effort of item writing was for the samples of teachers. Item clusters were prepared. These were revised into a form judged suitable for piloting. They were then piloted with groups of teachers, and revised a second time.

In all, a total of 660 items were prepared for the 'preference' surveys of teacher opinion. But the teachers taking part were required to grapple with only one cluster of items, a job which it was reckoned would take a typical respondent no more than half an hour. The clusters were, of course, 'rotated' to samples of respondents in order to obtain a comprehensive picture of the teachers' responses to all 660 items. This procedure had, itself, been previously tested for its validity and was not found to be wanting.

In addition to the one cluster of preference items that sought to elicit opinion, each form contained a small number of questions about demographic variables, and other questions that had been designed to stimulate thought about the future, particularly the 1980s. These questions were placed before the respondent encountered the preference cluster.

Surveys of preference were sent to samples of nine populations. Six of the nine were considered 'professional' populations, in that they were familiar with mathematics or with mathematics education. These six were:

AT: Mostly elementary teachers;
MT: Mostly secondary teachers;
JC: Teachers of mathematics at the junior college or technical school level;
MA: Mathematicians;
SP: Supervisors of school mathematics programmes;
TE: Teacher educators whose primary responsibilities are for mathematics education.

Samples of the first four of these populations were selected from lists of members of national organizations. This was done by choosing every 'n'th name. The number 'n' was calculated so as to ensure a sample of at least one hundred individuals responding to each cluster of items, assuming a modest rate of return of the forms. The SP and TE samples were randomly selected from lists of supervisors and teacher-educators prepared by the NCTM. These lists were not representative of the total cohort of individuals who make up the populations of supervisors and of teacher-educators. Rather it was believed to be best, for the purposes of the PRISM Project, to ascertain the views of those leaders in the schools who potentially have the ability to marshal support for new curricula.

Some clusters were designed expressly for the elementary school curriculum. Others were for the secondary school curriculum. Many clusters were considered applicable to both levels. The 'secondary' forms were used for the MT, JC and MA samples. The 'elementary' forms were used for the AT sample. The samples for the supervisors and teacher-educators presented two particular problems: the numbers involved were small, and many of the individuals concerned had responsibilities which span both the elementary and the secondary school curriculum. The decision was made to merge the items appearing under the categories headed resources, methods, and calculator into three generic clusters of items which were independent of the strands. This compromise alleviated both problems. And the generic clusters featured the alternative items that appeared on most of the clusters associated with the separate strands.

The other three of the nine populations sampled were considered to be 'laymen', so far as the issues and problems of mathematics education were concerned. A different pool of items, requiring no technical knowledge of mathematics, was constructed for these samples. Wherever possible, common items were used for both the lay and the professional samples. Special clusters were constructed. These focused on many of the more important issues, problems and alternatives that were embedded in the pool of professional items. The three 'lay' sample samples were:

PR: Principals of schools. A principal is the chief adminstrative official of a school building. In the United States, their background in teaching may be of any one of several different fields; only a few have extensive technical background in mathematics.

SB: Presidents of school boards. Each local school district is governed by an elected board of citizens whose primary responsibility is for policy, and for overseeing the administration of the schools. Few have technical expertise in mathematics education.

PT: Presidents of local parent-teacher organizations. The majority of schools have a parent-teacher organization. The parent-teacher organization has no official role in the operation of the schools, and is constituted almost exclusively of parents of children enrolled in the school. It represents a primary avenue of communication between the school and parents. Rarely will a member of a parent-teacher organization have technical expertise in mathematics education.

The preference surveys were posted to individual members of the samples during the period from September 1978 to March 1979; the actual time of posting depended on the population being sampled. Each survey form presented curricular alternatives that described practices associated with one of the nine strands. Furthermore, respondents were generally asked to consider alternative practices that were unique to one of the six categories. In other words, respondents were asked to concentrate on the cluster of items belonging to just one cell of the strand-category matrix. So, if the cluster concerned *methods* for teaching *algebra*, the respondent was requested to disregard other categories such as 'resources' or 'goals'. Thus, the items of the preference survey did not provide an opportunity to consider the interactive effect of such other categories as 'content' or 'methods' or the 'resources' used in teaching mathematics. The preferences sought were for the curricular alternatives associated with just one aspect of the teaching of just one broad, but important, area of mathematics.

The *priorities* survey was designed to seek opinion on curricular decision-making of a more global and interactive nature. And it was first thought possible to construct appropriate survey forms by making use of the preferences which the first survey had identified. Thus, a practice that was favoured from the cluster associated with *methods* in *geometry* might have been compared to a practice favoured in the cluster associated with *content* in *fractions and decimals.* This might have been done by asking respondents to indicate which of the two was the more important for attention and investment in development during the 1980s. However, this procedure did not prove to be a feasible way forward because of the time taken to process the data from the preference surveys, and the difficulties in agreeing upon an appropriate rationale for weighting the opinions of different populations.

Eventually, therefore, the priorities, or second-round, survey instruments were designed in such a way that respondents could indicate their order of preference for a variety of possible actions that could be taken to improve mathematics education, assuming the resources available for investment during the 1980s would be limited. Items were designed so as to force the respondent to choose his preference for possible goals, content, methods, resources, and attention to special categories of students. Other items sought an order of priorities in curriculum planning. A few items sought opinions on the effects of certain general problems rather than on problems specific to mathematics education. They also sought to elicit opinion on the relative priorities of the general problems as compared to the priorities of the problems specific to mathematics teaching and learning. An example of a general problem is student discipline. Some of the items on the survey of priorities asked individuals to indicate why they assigned top or bottom priority to a given curricular alternative. The priorities survey form concluded with questions on how to accomplish curricular change. These questions touched on in-service teacher education, large scale curriculum development projects, and research.

Four examples of items from the priorities instrument are given below:

Imagine that you have a limited amount of money to spend for the development of new materials for grades 7–12 in the areas listed below. Please indicate the order in which you think the money should be spent by marking the answer sheet in the following way:

a – highest priority
b – second highest priority
c – middle-level priority

d – second lowest priority
e – lowest priority

Be sure to use each letter *only once* for the next five items.

13. Algebra
14. Probability
15. Geometry
16. Computer literacy
17. Statistics

In your opinion, in what order of priority should the following areas within teacher education be addressed during the 1980s?

a – highest priority
b – second highest priority
c – middle-level priority
d – second lowest priority
e – lowest priority

Be sure to use each letter *only once* for the next five items.

42. Mathematics content
43. Methods for teaching mathematics
44. Development of teaching materials
45. Sensitivity to student needs
46. Diagnostic and remediation strategies

19. Consider the content area (questions 13 thru 17) above that you ranked highest (marked with an "a"). Of the following five ideas, which best describes the reason you gave it highest priority?

    a – We have fewer good materials to choose from in this area than in the other four areas.
    b – This is a major problem area for many, many teachers.
    c – It is absolutely crucial that more students develop skills in this area.
    d – New ideas have been developed in this area that are not reflected in present materials.
    e – The importance of this area will increase during the 1980s.

In previous questions you have ranked priorities within the broad areas of mathematics content, students with special needs, and teacher education. To these areas might also be added the development of non-text teaching materials and the development of special teaching methods. In what order should these areas be studied or developed during the 1980s? Please indicate your priorities by marking the answer sheet in the following way:

a – highest priority
b – second highest priority
c – middle-level priority
d – second lowest priority
e – lowest priority

47. Improved mathematics content for textbooks.
48. Development of special mathematics materials for students with special needs.
49. Improved pre-service and in-service education for teaching mathematics.
50. Development of non-text materials for teaching mathematics.
51. Improvement of methods and techniques for teaching mathematics.

Some Results of the PRISM Surveys

The surveys of the nine populations yielded information that would be useful in formulating recommendations for curriculum change and in planning the implementation of recommendations. The discussion that follows is focused on the few particular findings that best exemplify the potential usefulness of the information gathered.

Example 1. Problem Solving

The data from both surveys provided strong support in each of the nine populations for emphasizing problem solving. Indeed, in terms of goals for the mathematics curriculum, or for particular strands in mathematics, problem solving ranked first or second in every sample. In this context, it is of interest to remark that the Task Force of professionals who prepared the NCTM curriculum recommendations for mathematics for the 1980s elected to feature problem solving as the most important curricular area to be emphasized during the coming decade. This decision was made prior to the release of PRISM data to the Task Force. Nevertheless, the overwhelming endorsement of this decision among those surveyed served an exceedingly useful purpose. It meant that in preparing the rationale for their choice of the most important domain for curricular emphasis, the Task Force could concentrate on elaborations of their basic theme rather than on expending energy in justifying their choice of theme. Secondly, the choice for the basic theme did not need to be defended extensively against possible competing themes.

The fact that problem solving had been chosen as one of the primary strands of mathematics in the first-round preference surveys was fortunate. The choice was helpful, too, in revealing the sophistication of respondents concerning the teaching of problem solving. It emerged that the 'content' of problem solving is not considered in traditional curricular terms. Questions of scope and sequence are much more difficult to deal with than in other, more standard curricular domains like algebra. The item pool for the problem solving strand contained seventy items in six clusters relating to goals, content, resources, methods, who/time, and the use of the calculator in teaching problem solving. Collectively, these items served the purpose of indicating the level of sophistication of the respondents in weighing the issues raised by problem solving. In general it is evident from their responses that teachers are thoroughly conversant with heuristics and with other strategies used in problem solving. Indeed, the preferred emphases for instruction and content in the problem solving strand was for heuristics or comparable strategies.

To be able to appraise the level of sophistication of teachers about problem solving is a useful guide to the choice of language in which to frame the recommendation. It is also a useful pointer to the kind of materials to publish in implementation of the recommendations on problem solving. If materials are developed for the inservice training of teachers, or for the use of students in the classroom, the language, the concepts, and the approaches can be formulated in terms which are are consistent with the apparent understanding of teachers.

Example 2. Probability and Statistics

Mathematics educators and mathematicians have exhibited through a period of several decades a long-standing enchantment with probability and statistics as important content to be emphasized in the school mathematics curriculum. The rationale for the emphasis upon this strand of mathematics is generally based upon the argued need for an understanding of how to handle and interpret data. This, it is claimed, is necessary in order to function as a 'literate', and as an informed member of a scientific, technologcial society. The findings of the two rounds of surveys revealed a discrepancy between the views of the mathematicians, the supervisors and the teacher educators, and those of all of the other samples of populations on the importance of probability and statistics. Although the Recommendations for Mathematics in the 1980s do emphasize this strand of the mathematics curriculum, the PRISM findings suggest that special attention must be given to the plans for implementation before teachers, parents, and others will perceive the need for probability and statistics, or will accept a curriculum change with increased emphasis on the topic.

Example 3. Calculators in the classroom

The use of calculators in the mathematics classroom is quite controversial. For this reason, most of the strands in the preference survey had a cluster of items describing curricular alternatives which would make use of this tool for doing mathematics. The PRISM findings revealed a considerable discrepancy between the views of teacher educators and supervisors and those of the other samples. Indeed the discrepancy between the views of teachers and those of the supervisors and teacher educators is so marked that it needs to be considered whether the supervisors and teacher educators would be able to communicate with teachers on the issue in an in-service setting. The findings serve to warn NCTM of the potential problems of communication associated with advocating the extensive use of calculators in mathematics. Possible uses of the calculator that were described in the clusters of items in the calculator category included such practices as the use in tests, the use to do homework, the use to check the results of computation and the development of new concepts and skills. The most universally approved use of the calculator was for checking the results of computation. However, the level of acceptability rose in the context of more advanced levels of mathematics, such as trigonometry, or where use of a calculator would permit more complex problems to be set. Nevertheless the PRISM findings reveal both the extent and the nature of the communication problems which confront teacher educators.

Example 4. Reading and Mathematics

Several of the clusters of items concerned with goals of instruction for particular strands contained an item that stated that one of the reasons for giving instruction in the particular strand was to enable the students 'to learn how to read mathematics'. Items appearing in other clusters also specified curricular alternatives that emphasized teaching students how to read mathematics. Interestingly, although teachers indicate that learning to read mathematics is an important goal, the finding is not confirmed by analysis of the responses to items relating to methods and resources used for instruction. These suggest that teachers do not value the teaching of reading. The discrepancy between the attitudes to goals and the preferences in practices is puzzling. And the inconsistency may pose a problem of accommodation when curricular materials have to be prepared.

Example 5. Estimation

Estimation featured as a desirable skill in several of the clusters of items. Generally, all populations favour practices that emphasize estimation, though supervisors and teacher educators came out more clearly in support of these practices than the other populations did. However, examination of the most commonly used texts in the United States shows that these place little emphasis upon the topic. Thus, the PRISM findings reveal a discrepancy between what teachers and other professionals advocate and what the tools for instruction in the schools provide. Discrepancies of this type indicate a need for curricular development.

Example 6. Mechanisms for implementing curriculum change

The priorities survey invited opinion on how best to invest resources in order to promote better curricular practices. The majority of those sampled showed stronger support for investing in teacher education, particularly in the in-service variety, than in any other mechanism. This information provides NCTM, a professional organization, with strong evidence when it approaches government agencies, such as the National Science Foundation, for support for projects designed to implement the recommendations. The evidence also serves to guide NCTM in developing policies towards support of the recommendations and their implementation.

Example 7. Types of students and their needs

The educational system in the United States is designed to serve the interests of all types of students in elementary and secondary school. However, there have been periodic shifts of concern either for the talented or for the disadvantaged learners at different times throughout this century. The 1970s, for example, saw the greater part of the resources available for special categories of students invested in slow learners and the disadvantaged. The evidence of the PRISM surveys suggests that the trend now is toward a re-awakening of interest in the talented, and in providing special curricular alternatives to serve their needs. Thus, the PRISM data provide advance warning of a need to prepare materials and programmes for talented students, even though the evidence suggests that concern for the talented students has yet to reach its peak.

Example 8. Extremes in curricular practice

The findings showed little support among any population for unconventional alternatives in curricular practice. Although the majority of the items used in both surveys employed the kind of vocabulary that we believed would be understood by those being sampled, whenever alternative practices were described which were unconventional, they received scarcely a modicum of support from anyone. For example, it was proposed in one item that a calculator should be issued to any high school student (age fourteen) who had not learned to do numerical computations. The proposal was rejected. Lay people, teachers and all other populations showed a commitment to tested and familiar alternatives. Recommendations for curricular change in the United States must be implemented with an awareness of this characteristic conservatism of the groups who have a stake in the mathematics curriculum.

Example 9. Computer literacy

Each of the samples responding to items concerned with computer literacy showed a commitment to laying greater emphasis upon computer work in mathematics during the 1980s. It was as if the majority of respondents anticipate a future for students who, will in their adult life, be in a society that will make an even more dramatic use of, and be dependent on, the computer than is now the case. This outcome highlights the discrepancies that exist between the resources for instruction that are available in most school programmes and what is preferred. These discrepancies exist both in the hardware and in the software that is needed for instruction. Thus, the PRISM findings serve to identify a need for greater resources than are currently allocated to education in mathematics.

Summary and Conclusions

The NCTM PRISM Project was designed to collect information about the practices, and the priorities for curricular change, desired by a variety of populations who have a responsibility for, or an interest in, the effectiveness of school mathematics programmes. The findings have served to guide the formulation of NCTM's own recommendations for curriculum change for mathematics during the 1980s. More importantly, the PRISM findings have thrown up a number of problems of potential significance for NCTM, as it shapes policies, and plans activities, to promote the acceptance and implementation of its own recommendations. The information gathered by the PRISM Project provides NCTM with a more rational and systematic basis for planning curricular change.

Assessments of needs have traditionally examined discrepancies between opinion on what is and what ought to be. However, such discrepancies have most frequently been elaborated by contrasting performance in the schools' mathematics programmes with opinions about the appropriateness of goals and curricular content. The discrepancies brought to light in the PRISM findings go further. They provide evidence on the acceptability and support that may be expected for new curricular ventures. As a result, planning the implementation of recommendations can be done with greater precision, and with a more accurate assessment of the nature and the magnitude of the investment needed to produce curricular change. When the findings are contrasted with what is known about current practices in the schools, they indicate more precisely the nature of and the extent of the needs for development and in-service education. Of particular note is the bringing to light of the discrepancy between the value systems of teachers and of those who work with teachers, the supervisors and teacher educators.

Many of the procedures, techniques and processes used by the PRISM Project to collect information for the NCTM programme of curricular change could be adapted and used by other groups and agencies investing in projects aiming at producing change in the mathematics curriculum. Although the processes and techniques are expensive and time-consuming, the information gathered has the potential of decreasing the trauma of future curricular change. New curricula in the United States during the past twenty years have had a transitory, fleeting impact. Hopefully, the information gleaned by the PRISM Project will ensure the design of the strategies for implementation which will ensure more lasting change in the future than in the past – change that will consolidate, and build upon the modifications of curricular practice.

Murad Jurdak and Ed Jacobsen

The Evolution of Mathematics Curricula in the Arab States

The 1960s witnessed unprecedented changes in the teaching of mathematics, with nearly every country of the world introducing a new mathematics curriculum. The chapter in this volume by van der Blij, Hilding and Weinzweig describes these changes and the forces which were in effect. The Arab States were a part of this wordwide movement.

We must remember that many of the Arab States had, at that time, only recently become independent. Hence they had two pressing educational problems: the provision of mass education (at least at primary level) and the need for high-level manpower to develop their countries economically and technically.

Goals of Education and Mathematics Education in the Arab States

The goals of education in the Arab States are mostly derived, with different degrees of emphasis, from three main sources: the Islamic Arab culture, modern Arab nationalism and the needs and demands of social and economic development. The Arab States have tried, over the last thirty years, to unify, as much as possible, their education goals. In this respect, numerous regional conferences have been held. The most important of these formed two series: the conferences of the Arab Ministers of Education in 1953, 1964, 1968 and 1972; and the conferences of Ministers of Education and Ministers Responsible for Economic Planning in the Arab States in 1960, 1966, 1970 and 1977.

The thrust of the conferences of the Arab Ministers of Education was the promotion of unity and uniformity in the Arab education systems. The 1964 conference[1] approved the constitution of the then Arab League Educational, Cultural and Scientific Organization (ALESCO). It also recommended the standardization of the bases of school curricula in the Arab States. The 1968 conference[2] approved the unification of goals for general education, technical education and teacher training. The 1972 conference[3] focused its attention on the implementation of the previous recommendations, and defined the framework of a strategy for developing education in the Arab States.

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[1] League of Arab States. *The first and second conferences of the Arab Ministers of Education.* Cairo, 1953 and Baghdad, 1964. [In Arabic].

The concerns of the conferences of the Arab Ministers of Education and Ministers Responsible for Economic Planning were the promotion of economic and social development through education and within the heritage of Arab culture and national aspirations. The 1960 conference explored the possibilities of co-operation among the Arab States to meet their educational development needs. The 1966[4] conference discussed the goals of education in the Arab States and emphasized the quality of education. The conference of 1970[5] reviewed the main trends and future educational policies relating to the expansion of education for economic and social development and the improvement of the quality of education for developing human resources. The 1977 conference[6] focused its attention on the achievements made. It also considered the development of strategies that aim at providing alternatives to existing prospects for education development.

The goals of mathematics education in the Arab States are more or less similar. The third conference of the Arab Ministers of Education[7] approved, among other things, the goals of mathematics education in the Arab States. These goals centred on:

- attaining arithmetical literacy for all, and mathematical literacy and competence for the majority;
- preparing students for subsequent study or professions; and
- developing desirable habits and favourable attitudes.[8]

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[2] League of Arab States. *The third conference of the Arab ministers of Education.* Kuwait, 1968. [In Arabic].

[3] Arab League Educational, Cultural, and Scientific Organization (ALESCO). *Final report of the fourth conference of the Arab Ministers of Education.* Yemen, Sanna, 1972. [In Arabic].

[4] Unesco. *Final report of the conference of Ministers of Education and those responsible for economic planning in the Arab States.* Tripoli, 1966.

[5] Unesco. *Final report of the conference of Ministers of Education and those responsible for economic planning in the Arab States.* Marrakesh, 1970.

[6] Unesco. *Final report of the conference of Ministers of Education and those responsible for economic planning in the Arab States.* Abu-Dhabi, 1977.

[7] League of Arab States. *The third conference . . . op cit.*

[8] League of Arab States. *Final report of the committee for the study of unifying school curricula in Arab Countries.* Cairo, 1965. pp. 60-5 [In Arabic].

In the elementary school (ages 6 to 11 years), the acquisition of 'arithmetical literacy' involves the mastery of skills and the understanding of the fundamental operations on numbers and fractions. The goals also imply the understanding of basic ideas in geometry and of measurement and their application to everyday life. Arithmetical literacy is expected to reach the level of competence in the intermediate school (ages 12 to 14 years) and to be maintained thereafter. 'Mathematical literacy' is only aimed at tangentially in the elementary school. This is reflected by the two goals of 'acquiring quantitative thinking' and 'starting to use the logical method of inferences and expressions'.[9] 'Mathematical competence' is an important goal in the intermediate school, and in the secondary school (15 to 17 years).

'Preparing for subsequent study or for a profession' is an important goal for all three schools. Professions are included here because compulsory education in most Arab Countries does not extend beyond the elementary school.

The goals of mathematics education include also the development of habits and attitudes such as:

- being accurate and organized;
- acquiring an interest in mathematics;
- building self-confidence; and
- promoting sound moral and social attitudes.

An opportunity to implement these goals came almost as soon as they were formulated. In 1966, the 'Conference of Ministers of Education and Ministers Responsible for Economic Planning in the Arab States', meeting in Tripoli, called attention to the need for qualitative improvement in education and requested that emphasis be placed primarily on mathematics and science. Unesco was prepared to respond to this request, since, in 1961, Unesco had intensified its programmes aimed at stimulating improvements in the teaching of science and mathematics at pre-university level in the developing countries. Thus, in 1967, the Unesco Mathematics Project for the Arab States (UMPAS) was launched in order to assist these countries in their own efforts to improve the quality of mathematics education so as to produce, urgently, the needed scientists, engineers and technicians, as well as 'mathematically literate' citizens.

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[9] Ibid, p. 62.

The Unesco Mathematics Project for the Arab States

Three critical decisions were made in the planning stages of the project:

- the mathematics programme would be new, completely modern and based on a total reconstruction of the curriculum;
- the programme would be for the last three years of secondary education;
- the implementation would be through an international working group, national study groups and a committee composed of mathematics education from the region.

Let us examine each of these decisions in more detail. In 1966, most curriculum developers in mathematics felt that it was not possible to update the traditional mathematics programme simply by injecting some modern concepts and treatments. Rather, the key ideas of sets, relations and functions could be used to build a unified mathematics programme. An early project document described this approach as follows:

> 'Modernization means that the teaching of mathematics is based on a general attitude quite different from the one in traditional teaching. It proceeds, using the notions of set theory as a basis, to build up a more unified construction, structured by homogeneous ideas. Mathematics must be taught by stressing, at the appropriate time, the fundamental structures that occur in several branches of mathematics, both as means and as ends . . . . .[10]'

This view was reiterated in the first report of the committees and consultants engaged in the project, which stated:

> ' . . . . traditional syllabi are on the point of being abandoned in many countries, and determined efforts are being made towards the modernization of mathematics teaching at all levels and especially in secondary schools. In this process, it is being recognized that a radical departure from the traditional curriculum is a pre-requisite for any improvement and modernization.'[11]

The decision to introduce the programme into the last three secondary years of the 6-3-3 school structure, which most Arab countries favour, was based on several considerations. Despite the logical arguments for starting the reform in the primary school, it was felt that the quickest way to train high-level manpower was to concentrate initially on the secondary programme, and especially on the pupils of high ability. In the second place, it was known that secondary school teachers were far more capable of adjusting to new programmes than were elementary school teachers, who frequently had meagre mathematical training. And, while it was recognized that the retraining of secondary school teachers would be a difficult task, it was evident that the retraining of a far larger number of elementary school teachers would be far more difficult. The project document quoted above, after suggesting an outline of the topics to be included in the mathematics programme, goes on to say:

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[10] Unesco. *Unesco Pilot Project for the Improvement of Mathematics Teaching in the Arab States.* Paris, Unesco, 1966 (WS/1266/101/AVS/DST) Appendix II.

[11] Unesco. *School Mathematics in Arab Countries.* Unesco Mathematics Project for the Arab States. (UMPAS) Paris, Unesco. (SC/WS/201) January, 1969.

> 'The same programme taken at a slower pace, less deeply and with many applications could very well be the *curriculum for all pupils*. It is possible to arrange a sequence of essentials for the *less capable* or the less prepared, so as to enable them to understand some mathematics regardless of the stage at which they have ceased their formal study.'[12]

The implementation of the project utilized the following structure:

- National Study Groups for the Modernization of Mathematical Education;
- An International Working Group;
- The Inter-Arab Commission on Mathematics;
- Regional Seminars.

National Study Groups (NSG) were formed in each of the participating countries two years before the pilot project was launched. The advantage of having NSGs had been shown from earlier Unesco pilot projects in other regions.[13] As their name implies, their function was to examine the situation in their country in order to make recommendations in the International Working Group on the content and the approaches which they would like to see implemented. But they were intended also to be permanent bodies, responsible for planning and implementing the mathematics reform in their respective countries, and were to represent all segments of the mathematics community in the country. Thus, NSGs were established in 1967 in the Sudan, the United Arab Republic, Iraq, Jordan, the Syrian Arab Republic and Bahrain and by the United Nations Relief and Works Agency for Palestine Refugees in the Near East (UNRWA). Unesco provided the NSGs with materials from earlier curriculum reform projects. These laid the groundwork for the creative task which the regional pilot project participants would eventually have to do.

[12] Ibid., Annex II, p. 2.

[13] Baez, Albert V. *Innovation in Science Education – World-wide*. Paris, The Unesco Press, 1976. 249 p.

The International Working Group (IWG), designed to function for only one year, was made up of internationally recruited mathematics education specialists, together with a number of Arab mathematicians who had had outstanding teaching experience at university or teacher education levels. The first task of IWG was to organize the regional seminar, held in 1969, which prepared the detailed syllabuses for the secondary schools and developed plans for writing and evaluating the books. Their report[14] contained guide-lines for an experimental syllabus, with suggestions on vocabulary, arrangements of material, tests, exercises and procedures for implementing the innovations. The report also recommended the establishment of an Inter-Arab Commission on Mathematics. The task of this Commission was to encourage research and the development of mathematics and of mathematics education in the region.

Since much of the work of the Unesco project was to implement the recommendations of the IGW seminar, we now list some of them:

- The proposed course is intended for students who will confine themselves to scientific sections and scientific careers. It is left to each country to select from it, and to decide on the appropriate approach and content for the non-scientific streams;
- Methods of teaching are crucial to the implementation of the new programme. Hence, we expect textbook writers and teachers to present the materials in the spirit of the programme. In such presentation, students are expected to participate actively in developing the ideas and concepts, moving from intuition to generalization and from generalization to abstraction;
- It is to be assumed that some of the students involved will be college bound. For these, an appropriate level of abstraction must be sought. Some, however, will not continue their education in a university. Hence, applications of mathematics must be an integrated part of the material.
- Arab States should adopt a syllabus which reflects the spirit of contemporary mathematics. The syllabus should include:
  - a) Sets, relations, functions (or mappings);
  - b) Algebraic structures and their realizations in the number systems;
  - c) Geometric structures, transformations, co-ordinates, vectors;
  - d) Logic and proof;

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[14] Unesco. *Regional Seminar of the Unesco Mathematics Project for the Arab States, Cairo.* 1969 (SC/WS/227) (The entire report was also printed in Arabic in: *Journal of Education* Vol. XXI, No. 4 (May 1969), association of the Graduates of the Institutes and Faculties of Education, Cairo, p. 1–100).

e) Probability, and mathematical statistics;
f) Elements of analysis, and some topological notions;
g) Applications, mathematical models.

- The content must include such topics which will be needed to serve mathematics and other subjects as well. (Mechanics was considered a national option);
- Applications in both mathematics and other areas of human activity must be considered. Hence, numerical analysis and work with approximate values have to be included whenever possible.
- An axiomatic approach must be adopted in order to give secondary school students the chance of grasping mathematics as a structure in which they can see and appreciate the meaning and the role of undefined terms, definitions, postulates and theorems, along with the need for proper methods of reasoning which help them to derive and build on more knowledge.

In 1969-70, four writing sessions were held: Baghdad and Beirut in 1969; Amman and Kuwait in 1970. These sessions prepared drafts of experimental texts for grades 10 to 12. Unesco printed them in English and translated them into Arabic. The grade 10 book was printed in 1970, and trials of the book starting that year in Kuwait, in the Syrian Arab Republic and the then United Arab Republic. Trials were extended to other countries the next year. Coutries were free to make their own final revisions, and to produce copies as required. For the first time, all books used the same Arabic mathematical terminology.

After the initial writing, the story becomes rather complicated, as each country proceeded in its own manner to revise the textbook and try it out. Some details are provided in a later section.

In-service Courses.

During the initial phase, month-long regional training seminars were held (Damascus in 1970, Tunis in 1971, Cairo in 1972) to familiarize the pilot project teachers with the new textbooks. In each seminar, teachers from fourteen Arab countries participated. The objectives of the seminars were to extend the mathematics background of the participants by emphasizing topics basic to a modern programme in mathematics for secondary schools.

Unesco's main involvement in the project terminated with the convening of three seminars. An Evaluation and Planning Seminar (Nice, 1970) examined the problems of the experimental classes and recommended that the three-year intermediate school mathematics programme should be reformed in harmony with the new programme for the secondary schools. At the Unesco Regional Seminar on Science

Education Improvement (Cairo, 1971), there were five working groups, one for each school science subject. The recommendations[15] concerning mathematics were:

- to start a project for the preparation of mathematics textbooks for grades 1–9;
- to co-ordinate mathematics teaching with other school subjects (in particular, the role of applications); and
- to establish and finance the Inter-Arab Committee on Mathematics previously recommended in 1969.

The Seminar on the Changing Role of the Mathematics Teacher (Cairo, 1974) gave detailed recommendations[16] on the education of teachers for primary and secondary levels.

The ALECSO Follow-up

The Arab Organization for Education, Culture, and Science (ALECSO) has, since 1972, given special consideration to the reform of mathematics teaching in the Arab States. It has promoted experimental teaching with the textbooks developed by the Unesco Arab Mathematics Project, as well as implementing a mathematics project of its own for the intermediate school. ALECSO has made possible a most valuable exchange of ideas between the Arab States. During the summer of 1975, they organized five teacher training seminars for secondary school teachers, and one seminar for teachers experimenting with the ALECSO textbooks for intermediate schools in Cairo.[17]

ALECSO has also convened an evaluation seminar, which, after reviewing the situation concerning mathematics education in the region, felt that the syllabus and textbooks were at too high a level for the majority of the students. In particular, the textbooks were considered to be too theoretical and contained too few examples, and the chapters were not integrated (having been written by different authors without provision for one editor with final editing responsibilities).[18] A similar view was expressed in a report to Unesco:

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[15] Unesco. *Regional Seminar on Science Education Improvement (Cairo, 1971)*. Paris, Unesco 1971 (SCT/DST/6101/56) pp. 101.

[16] Unesco. *The Changing Role of the Mathematics Teacher: Report of a Seminar held in Cairo UAR, 29 Sept. – 8 Oct. 1974*. Paris, Unesco, 1975. (ED-74/CONF./41).

[17] ALECSO. *Training Courses for Secondary and Intermediate School Teachers: Mathematics.* Cairo, ALECSO, 1975.

[18] Hastad, Mats. *Seminar for the Evaluation of the Mathematics Project in Secondary Schools in Arab Countries (Damascus, Syria 5 – 10 July 1975)*. Paris, Unesco, 1975, pp. 11.

> 'It is seen that, although the curriculum is well integrated and follows a suitable sequence, it is beyond the ability of the average student. This is especially the case when we take into consideration the fact that education in most Arab countries does not allow for individual differences. Every student at the same grade has to take the same syllabus at the same level and at the same period of time.'[19]

These criticisms were probably valid, especially as the books were aimed primarily at those who would continue their mathematical studies at university level. In reality the ability range of the students was much broader.

Eventually ALECSO decided to revise the Unesco secondary schools books. At a meeting in Alexandria in 1976, a three-year course was planned, based upon the ALECSO curriculum for the intermediate stage. After holding a series of writing workshops, books for grades 10–12 were published successively in 1977, 1978 and 1979. Further meetings were organized by ALECSO in 1978. Authors of the grade 11 textbook met in Khartoum to review the preliminary version and to plan the contents of the twelfth grade text. In Amman, an evaluation seminar was organized to prepare tests and other possible instruments to evaluate student achievement in mathematics in grades 7 to 9. This seminar also began planning for an examination system in mathematics which would take account of the developments which had occurred in the curriculum and in the teaching methods.[20]

The Situation To-day

Mathematics in the total curriculum

Compared to other subjects, mathematics is given a substantial weight in the total school curriculum. This is reflected in the high percentage of the number of weekly periods allotted to mathematics. In the elementary school, mathematics is allotted 15 to 21 per cent of the usual thirty teaching periods per week of 40 to 50 minutes each. It is worth noticing that mathematics ranks third (after Arabic language and religion) in terms of time allotment. In the intermediate school, the percentage varies between 13 and 17 per cent of the thirty to forty weekly periods of 50 to 60 minutes each. And in the secondary school, the percentage varies between 16 and 26 per cent of the 30 to 42 weekly periods of 55 to 60 minutes each.

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[19] Abou-Yousef, M.I. Consultant's report on the Seminar for Constructing Means of Measuring the ALECSO Mathematics Curriculum for the Intermediate Stage (Amman, 9 – 18 April 1978), Paris, Unesco, 1978, (restricted).

[20] Ibid.

The weight given to mathematics has not appreciably changed since the survey of 1969. Such weight is still comparable to that of other developed countries.

Subject matter

Elementary school.

In general, the main changes in mathematical content in the elementary school have involved the placement, the sequence and the organization of topics. The mathematical content, in most cases, still consists of a study of the natural numbers and of fractions (common and decimal), with the four fundamental operations and their associated algorithms, together with the elements of non-metric geometry and the basics of measurement. However, a trend towards greater mathematical literacy seems to be gaining momentum. First, more emphasis is placed on mathematical operations and on the steps which lead to the algorithms. Second, concepts in non-metric geometry are gradually introduced right from the start. Third, the approach to measurement is becoming more and more integrated with geometry and with numbers in the sense that measurement systems are not only dealt with from the utilitarian point of view, but also as models of the metric system. These trends seem to be distinct changes as compared to the situation described in the 1969 survey of school mathematics in the Arab countries.[21]

Many national efforts in which one or more of these trends are reflected have been started or completed. Iraq has experimented with and has completed a revision of the objectives and of the organization of its elementary mathematics programme. The adoption at the national level of these reforms for all the six elementary grades was completed in 1976/77, after six years of experiment. Sudan has revised its programme in elementary mathematics, and the new curriculum is expected to be fully implemented by 1981, after six years of experiment. Saudi Arabia has adopted a fairly 'activity-orientated' elementary mathematics programme. New textbooks were prepared, and these are being implemented and evaluated concurrently. It is expected that the process of implementation, evaluation and revision will be completed by 1982. Lebanon adopted an up-dated elementary mathematics programme in 1971; its implementation was completed by 1974. Jordan and the Syrian Arab Republic started a revision of their elementary mathematics programme as part of their attempts to unify the curricula of the two countries. The implementation of the programme is expected to be completed by 1982. Egypt is in the process of revising all school curricula. This involves some additions, some deletions, and changes in the sequence and placement of topics.

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[21] Unesco. *School Mathematics in Arab Countries*, op. cit.

Intermediate school.

In many Arab States, the mathematical content of the curriculum of the intermediate school has experienced some changes. Some of the aspects of the revised programmes are:

- the introduction of set language, and of such unifying concepts as relations, functions or mappings;
- a more systematic construction of the integers and of the rational numbers; and
- the use of different approaches to geometry.

In some cases, the ALECSO Mathematics Project for the Intermediate School has been the starting point. In others, countries have started their own national efforts. In 1974/75, Iraq started the adaptation of the ALECSO programme on an experimental basis. After modification, to fit the Iraqi situation, the programme was implemented in all schools in 1977-78. Jordan started experimenting with a revised intermediate mathematics programme in 1975. This programme will be fully implemented by 1982. A new programme, incorporating the transformation approach to geometry, and a systematic development of number systems by the extension method, is being implemented, and evaluated, in Saudi Arabia. Full implentation of this programme is expected to be completed by 1982. An up-dated programme has been implemented in all intermediate schools in the Sudan after three years of experiment.

Secondary school.

UMPAS has been a significant point of departure for revising the mathematical content of the secondary school curriculum. Unifying concepts are generally becoming part of the curriculum. Algebraic structures are increasingly being used to develop number systems. Innovations include elementary linear algebra, transformation, vector and co-ordinate approaches to geometry, probability and mathematical statistics, and the language of sets and logic. Calculus is increasingly being taught in the scientific streams of the secondary school.

Arab countries have either experimented with the Unesco project and later adopted it, or are still adapting it and up-dating their present curricula. Jordan, Kuwait and the Syrian Arab Republic belong to the first category; Iraq, Sudan, Saudi Arabia and Qatar belong to the second.

All in all, the mathematical content of school curricula is becoming more and more in line with contemporary trends.

Methods of teaching

There is little evidence to indicate that methods of teaching mathematics have changed significantly in the last fifteen years. Although the revision and reorganization of the subject matter has potentially made mathematics more meaningful, the methods of lecturing and of rote learning are still dominant. The utilization in the elementary school of 'child-centred activity learning' and of manipulative materials is the exception rather than the rule. However, familiarity with contemporary mathematical concepts is on the increase among secondary teachers.

It has been noted that a significant effort has been made to upgrade the mathematical knowledge of intermediate and secondary teachers. Short, intensive training programmes have been organized, usually before the introduction of a revised curriculum. However, this *ad hoc* in-service training has not developed into institutions to improve systematically the preparation of teachers in mathematics and its pedagogy. It is, however, relevant to mention that universities and colleges have in general strengthened their mathematics offerings.

Textbooks

Many changes have taken place in the purpose and in the appearance of mathematics textbooks. Textbooks are better illustrated and printed. Teacher guides are more commonly available now than they were ten years ago, although they are still limited in the scope and degree of guidance they provide for teachers. Mathematics textbooks have not yet been fully transformed to become learning tools for the self-study of students. However, in the elementary school, mathematics textbooks are increasingly playing the role of a work book rather than that of a textbook.

Examinations

One major handicap to curriculum change in the Arab countries is the national examination system National examinations play a central role in the future of the students. This is because at the end of every cycle, a student has to pass the national examination in order to move into the next cycle. The examination in mathematics usually covers the work done in the last year of the cycle, and it samples questions which call mostly for lower cognitive processes. Thus the examination itself promotes memorization and rote learning.

There is a wide-spread dissatisfaction with the examination systems in the Arab States. Some Arab countries have begun the process of reform. Egypt is one – the aim of the Egyptian project is to move gradually towards more objective tests of the total curriculum which will serve both diagnostic and formative purposes[22]. Saudi Arabia, also, is currently evaluating its examination system.

Process of curriculum development

UMPAS has affected the process of curriculum development in three respects. First, it provided a successful model of co-operation among the Arab Countries in curriculum development. Second, it pioneered the National Study Group. This brought together the expertise and experience of university professors, educators and teachers in a concerted effort of curriculum-making. Third, it offered a concrete example of a complete cycle of curriculum development – a cycle of designing, testing and revising the curriculum, before full implementation. The process followed in the Unesco project is markedly in contrast with the procedure, adopted all too often, of assigning to a committee the job of evolving the syllabus outline, and to an individual the job of writing a textbook for immediate full implementation. The model provided by the Unesco project led the 1978 Regional Seminar on Curriculum Reform in the Arab countries to recommend the holding of a workshop on 'a new strategy for curriculum development, using the experience of modern mathematics'.[23]

An indication of the growing role of research in the development of curricula is the increasing number of research and development centres which deal, exclusively or partly, with mathematics curricula. Among these are: the Curriculum Research Centre in Kuwait, the National Centre for Educational Research in Egypt, the National Institute for Improving Mathematics Education in Iraq, the Bakht-er-Ruda Institute of Education in the Sudan, the Centre for Educational Research and Development in Lebanon, and the Science and Mathematics Education Centre (SMEC) at the American University of Beirut.

Summary and Comments

Educational goals in the Arab countries are increasingly being geared towards social and economic development within the Arab cultural heritage and the national aspirations. The rapid expansion of the educational systems in the Arab States has resulted in a flow of students with diverse abilities and backgrounds. This presents new pressures and poses economic and pedagogical difficulties. The policy of most Arab countries is to extend compulsory education beyond the elementary school. However, the pedagogical difficulties result in rather high percentages of repetitions and of drop-outs. Mathematics is viewed as an important subject of the curriculum, which serves the social and economic development of the country. The content of school mathematics in the Arab countries has evolved considerably in the last fifteen years in response to contemporary trends. The teaching methods, and the training of teachers, have not kept up with the changes. The examination system has not evolved with the curriculum. It is still a major influence, and to it the content and the methods of mathematics are geared. The process of curriculum development is becoming more and more experimentally oriented, with a wider participation of mathematicians, teachers and students. The growing number of educational research and development centres is evidence of this trend.

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[22] Egypt. National Centre for Educational Research. *Projects and achievements of the National Centre for Educational Research in 1975–1976.* Cairo, 1977. [In Arabic].

[23] Regional Office for Education in the Arab Countries. *Final report and recommendations of the Regional Seminar on Curriculum Reform in the Arab Countries.* Cairo, 1978. [In Arabic] p. 12.

In the closing paragraphs, which follow, a few problems will be identified, and comments will be made on possible new directions and trends.

There is a need to translate the goals of teaching mathematics into operationally-defined objectives. If mathematics is to contribute to 'rural development', for example, relevant objectives have to be derived. These should be prescribed for the levels concerned, including, of course, the level of the classroom. In this way, it should be possible to determine the objectives (the concepts, the skills and the attitudes), the mathematical content, the appropriate methods of teaching and the appropriate process of evaluation which would eventually serve the goal of 'rural development'. It is relevant to mention here that the statements of goals of teaching mathematics have not changed, even though the mathematical content has in many cases been overhauled.

Reforming the content of school mathematics does not guarantee any reform of school mathematics curricula. A synchronized effort in teacher preparation and in the development of appropriate evaluation techniques has to accompany the up-dating of the content. The Arab countries find many obstacles in the way of providing the needed teacher preparation in the scope that is desired and at the level that is desired. In most Arab countries, the reform of school mathematics at the elementary level (which represents the largest base for their educational systems) is supposed to have top priority. However, countries with limited resources and competing priorities are discouraged by the large numbers of inadequately prepared elementary teachers from starting any long term plan for upgrading the professional competence of elementary teachers. Other Arab countries face the problem of attracting and retaining qualified individuals in the mathematics teaching profession. The *ad hoc*, single shot intensive training sessions are of limited impact if institutions are not established for this purpose of making teachers 'life-long students of mathematics and pedagogy'. It seems that more time and effort need to be invested if teachers are to be enabled to change their established practices and patterns of teaching behaviour. The remark made in the report on school mathematics in the Arab countries is still relevant and valid: 'It goes without saying that the greatest facility that is hoped to be achieved by using terms such as sets, functions, operation, relations, etc. is of no value whatsoever, if these very subjects are approached by the traditional methods, relying solely on memorization.'[24] Dissatisfaction with the examination system is prevalent among school teachers, university teachers, students and the community at large. Efforts to reform present systems have to be accelerated and extended if curriculum reform is to have any lasting impact.

One discernible impact of UMPAS is its contribution to the improvement of the curriculum development process. Experimenting with other curricula than that of mathematics is gaining ground in the Arab countries. However, 'experimenting', as it is applied, involves a wide range of interpretations. They range from informal visits to classes over a short period of time with a view to offering informal comments to a long-term process of evaluation with proper controls and valid instruments. It is not unusual to find that decisions to adopt a curriculum are sometimes made on the outcomes of faulty experiments. Greater efforts have to be made to establish and to institutionalize experiment as an on-going research activity, with a valid theoretical basis and with proper, in-built controls.

The mathematics curriculum in the Arab States has to face the challenges posed by the new types of schools that are being tried in some Arab countries. Egypt is experimenting with the comprehensive school, and with the polytechnic school. Several other Arab countries have started similar experiments. The Sudan is experimenting with the 'Integrated Rural Education Centre', which is a new type of elementary school, geared towards rural education and providing also informal adult education. New mathematics curricula of appropriate content, organization and approach have to be developed to serve the goals of these new types of school.

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[24] Unesco. *School Mathematics in Arab Countries.* op. cit., p. 89.

David F. Robitaille

Goals of the Mathematics Curriculum in British Columbia: Intended, Implemented and Realized

Introduction

In examining the goals of the mathematics curriculum of any jurisdiction it is important to bear in mind that the implications of and the meaning of those goals may, in large measure, be dependent upon the perspective of the observer. It may be the case that the goals will be understood in quite different ways by curriculum developers and administrators than by teachers, and differently again by students than by either teachers or curriculum developers.

The need to be aware of this divergence of points of view is a pressing one. If discussions and comparisons of mathematics curricula are to have any meaning, or lasting impact, they must be founded in fact. It would be inappropriate to reach conclusions about the goals of the mathematics curriculum solely, or even mainly, on the basis of what one thinks, believes or hopes the reality of the situation to be. On the contrary, steps must be taken to discover what the situation in schools and classrooms actually is.

The goals of the mathematics curriculum may be considered at any one of three levels: intended, implemented and realized. The 'intended' goals are those promulgated by curriculum developers. They are set out in the teachers' editions of textbooks, and are listed in curriculum guides. The 'implemented' goals are the goals of the curriculum as they are understood and implemented by teachers in their classrooms. Finally, the 'realized' goals are those attained by the students, as demonstrated by their performance on tests designed to measure mastery of the 'intended' goals.

The Ministry of Education of the Province of British Columbia (B.C.), Canada, embarked upon its most recent revision of the mathematics curriculum for Grades 1 through 12 in the early 1970s, and the process has continued over several years. The purpose of this paper is to examine the goals of that revised curriculum at the three levels: as intended by the Ministry of Education and its Curriculum Revision Committee; as implemented by teachers of mathematics throughout the province; and as realized by their students. It is hoped that such an exposition will serve to emphasize the importance of looking at matters which affect the teaching and learning of mathematics, not from a subjective perspective, but from a more nearly objective one.

In order to establish a context for the discussion which follows, a brief outline of the education system in British Columbia and of its mathematics curriculum is given below. This is followed by an examination of the goals of that mathematics curriculum from the three perspectives mentioned earlier. Data obtained from two major projects[1], undertaken recently by the author on behalf of the Ministry of Education, will be used to illustrate the current status of those goals as 'intended', as 'implemented', and as 'realized'.

The Education System in British Columbia

British Columbia, like each of the other provinces in Canada, has complete autonomy over educational matters within its boundaries. Over the years, the education systems of the several provinces have evolved separately along more or less parallel lines. Although the systems are similar in any respects, they do differ in some significant ways as well. There is, for example, no unanimity among the provinces on such diverse matters as the age of initial entry into school, the number of years of schooling required to complete secondary school-leaving requirements, the existence of centralized systems of examinations and the provision of financial support to independent schools. Moreover, each province determines its own curricula in all subjects; there is no nation-wide body with a mandate to co-ordinate such developments. Each province acts virtually independently of the others insofar as educational matters are concerned.

The B.C. school system consists of kindergarten, and Grades 1 through 12. Students enter Grade 1 at the age of six. Kindergarten classes for 5-year olds, and Grades 1 through 7, are housed in elementary schools. Grades 8 through 12 are housed in secondary schools. Attendance at school is compulsory for all children under the age of 16.

At present, students are required to take a mathematics course each year up to and including Grade 10[2]. According to the official curriculum guide issued by the Ministry of Education, all students below Grade 11 are expected to follow the same basic curriculum in mathematics. No special courses or programmes for exceptional students, gifted or otherwise, have been officially defined.

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[1] Both projects, the B.C. Mathematics Assessment and the Mathematics Achievement Test Project, were funded under contract by the Ministry of Education, Province of British Columbia.

[2] A decision to amend this to include Grade 11 also was recently announced.

Although there is in B.C. no formal, centralized system of examinations to evaluate the achievement of individual students, the Ministry of Education does collect data on student achievement. The Learning Assessment Branch each year conducts an assessment of student achievement in one or two areas of the curriculum. The major function of these assessments is to provide information to administrators, curriculum developers, teacher educators, researchers, teachers, and the public at large about the degree to which the goals of education in those subject areas are being attained. Results are published for the province as a whole. No individual student, class or school results are made public.

During the 1976-77 school year, an assessment of mathematics was conducted. All students in Grades 4, 8 and 12 took achievement tests constructed specially for the study. In addition, approximately 3,000 teachers of mathematics in Grades 1, 3, 5, 7, 8, 10 and 12 completed detailed questionnaires dealing with instructional practices and the goals of the mathematics curriculum.

In 1978, a series of tests entitled *Mathematics Achievement Tests (MAT)* was constructed under the auspices of the Learning Assessment Branch for use mainly by teachers of mathematics in Grades 3, 7 and 10. Each item on these tests reflected an objective taken from the provincial curriculum guide for mathematics, and the tests were intended to be used by teachers at their discretion to assist in evaluating the achievement of their students.

The Intended Goals

In the preliminary draft of the revised curriculum guide for mathematics for Grades 1–3 and 7–8, the need for a major re-thinking of the mathematics curriculum was described in the following way:

> During the late 1950's and through the 1960's reform was largely in the hands of pure mathematicians. Although many gains have resulted from such reform, there has developed a lack of relevancy of this rigorous approach for a large segment of the population. Mathematics programmes are being developed to take into account the changing needs of the students.[3]

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[3] Division of Instructional Services. *Mathematics: Primary, Years 1–3, Years 7–8. (Preliminary draft for information).* Victoria (B.C.), Department of Education, 1972, p. 2.

The revised curriculum was founded on the assumption that, in Grades 1–10, all students should follow the same basic programme in mathematics. No 'streams' or 'tracks', based upon the students' mathematical abilities, were defined; for each grade a list of content objectives was given. Teachers were encouraged to use their discretion in interpreting the precise meaning of these objectives, and in implementing them. They were also advised to consider these lists of objectives as being prescriptive, but not exhaustive.[4]

The curriculum guide included a statement of the major cognitive goals of the revised curriculum. Their intention was to enable the student:

- to identify and to use the basic properties and operations of the real number system,
- to identify common goemetric figures, and to demonstrate a knowledge of their basic properties;
- to transform given numerical and algebraic expressions into equivalent algebraic expressions;
- to solve open sentences of various types and of degrees of complexity,
- to apply knowledge of mathematics to familiar physical or environmental situations in order to construct a descriptive mathematical model of the situation, or to solve a problem arising from the situation.[5]

In the new programme, increased emphasis was to be given to geometry in all grades. The geometry to be taught was to be informal and intuitive, rather than formal and axiomatic. There was also to be an increased degree of emphasis on the applications of mathematics in the real world. This emphasis was illustrated most clearly by the inclusion of a major unit, entitled 'Consumer Mathematics', as part of the Grade 10 course, along with more customary units on algebra and geometry.

The Implemented Goals

Analysis of questionnaires from the Mathematics Assessment Project indicates that the typical teacher of mathematics at the elementary school level in British Columbia has had slightly more than four years of post-secondary education and eight and a half years of teaching experience. About 75 per cent have studied some mathematics at the post-secondary level. Over 85 per cent have had professional training in the teaching of mathematics. The typical teacher of secondary mathematics has had slightly more than five years of post-secondary education. About 65 per cent had mathematics as one of their major areas of study as undergraduate students, and 80 per cent have had professional training in methods of teaching mathematics.

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[4] Curriculum Development Branch. *Mathematics Curriculum Guide: Years One to Twelve.* Victoria (B.C.), Ministry of Education, 1978, page 6.

[5] Ibid., p. 6.

In one section of the questionnaire, teachers were asked to rate the importance of a number of content objectives selected from the list contained in the curriculum guide for their particular grade. Across all grades, about 90 per cent of the objectives were rated positively; in other words, few were seen as being unimportant. All of the objectives relating to the acquisition of computational skills were considered to be important by all groups of teachers. On the other hand, the geometry objectives were consistently ranked in the bottom third of the list of objectives for each grade.

Teachers were also asked to rate the importance of certain curricular objectives for students completing secondary school. Among the ones rated as most important were those concerning computational skills, and the ability of students to apply their knowledge of mathematics in other areas, including that of Consumer Mathematics. These results are in line with the spirit of the revised curriculum as expressed in the curriculum guide.

When secondary teachers were asked to rate the importance of a number of objectives for students completing elementary school, they came out strongly in favour of emphasizing computational skills. Notwithstanding the emphasis upon geometry in the curriculum guide, teachers of both elementary and secondary mathematics judged geometry to be considerably less important than most other topics in the curriculum.

While the curriculum guide does not attempt to prescribe the methodology to be used in the teaching of mathematics, it does include a number of suggestions and recommendations on such methods. For example, teachers are reminded that their students should be given 'freedom to explore' and that 'before any mathematics can be understood, there must be a wealth of manipulative experiences through which concepts and applications are understood.'[6] Regarding the use of textbooks, 'it is strongly emphasized that no single text be regarded as the sole instrument for meeting the objectives of the mathematics program.'[7]

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[6] Ibid., p. 1.

[7] Ibid., p. 3.

There is no evidence from the data supplied by the questionnaire to indicate that teachers in British Columbia, particularly at the secondary school level, have changed their teaching approaches and adopted such practices as discovery teaching, individualized instruction, use of activity centres or projects to any significant degree. Moreover, only a small proportion of respondents indicated that they made any substantial use of audio-visual aids or any other teaching aids. The data showed that the most frequently-occurring activities in secondary mathematics classrooms in the province are individual classwork, work on textbook exercises and teacher explanation or demonstration. Regarding the use of textbooks, more than 75 per cent of the teachers surveyed said that they used either one textbook exclusively or multiple texts, with the use of one being predominant. Few have adopted the 'multi-text approach' suggested by the Ministry.

Another area in which there is a considerable gap between the 'intended' and the 'implemented' goals of the curriculum concerns the introduction of the metric system (SI) of measurement. British Columbia, along with the rest of Canada, is in the process of changing from the use of the Imperial system of weights and measures to SI. Only textbooks which are completely metric (i.e. they use only metric units of measure) are authorized for use in the schools of the province, and the curriculum guide has for several years contained a statement to the effect that the metric system should gradually supplant the Imperial system, with the metric system to be used exclusively from 1978. In response to an item on the questionnaire, the majority of teachers stated that they did not adhere to this directive, but continued to use both systems in their teaching.

The curriculum guide also recommends the use of hand-held calculators and computers 'in imaginative ways to reinforce learning and to motivate the students . . . .'[8] Of those teachers who agreed that calculators should be used in mathematics classes at some grade level, 50 per cent of those in Grade 8 and 25 per cent of those in Grade 10 said that they personally did not make use of hand-held calculators in their teaching. On the other hand, almost all teachers of Grade 12 mathematics said that they allowed their students to use calculators. Of those who agreed that computers should be used in mathematics classes, 70 per cent of those from Grade 8 and 10 and 50 per cent from Grade 12 stated that they did not do so themselves. Of those teachers in schools where a computer was actually available for use by students, approximately 60 per cent of the Grade 8 and 10 teachers and 50 per cent of the Grade 12 teachers indicated that they made no use of computers in their own mathematics classes.

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[8] Curriculum Development Branch., op. cit., p. 29.

Additional evidence of the existence of discrepancies between the 'intended' and the 'implemented' goals was obtained in connection with the MAT Project. As part of the pilot-testing of test items for that project, teachers were asked to examine sets of items from the item pool for their grade level and to indicate *whether or not they had taught the content involved in each of those items to their students.* Since each item had been designed to test student attainment of an objective from the prescribed curriculum, it was not unreasonable to expect that teachers would indicate that they had done so.

For certain objectives this was certainly not the case. At the Grade 10 level, for example, the curriculum guide includes a brief unit on the topic of vectors. Over 60 per cent of the teachers stated that they did not teach that content. Similarly, over 50 per cent indicated that they did not teach that part of the unit on Consumer Mathematics which dealt with the topic of compound interest. As a matter of fact, a significant proportion of the Grade 10 teachers indicated that they taught very little of either the Consumer Mathematics unit or of the geometry unit.

Similar discrepancies were found at the Grade 3 and Grade 7 levels, where the pilot-testing of items was also carried out. Over 30 per cent of the Grade 3 teachers said that they did not teach topics in measurement such as the metric units of mass, volume or capacity. Nor did they teach topics in geometry such as the classification of 3-dimensional shapes and lines of symmetry. Over 25 per cent said that they did not teach students to round off numbers to the nearest ten or hundred. Comparable numbers of Grade 7 teachers indicated that they did not teach some of the prescribed geometry content and graphing techniques. About 20 per cent said that they did not teach several of the metric units of measurement prescribed in the curriculum guide.

The Realized Goals

Both the Mathematics Assessment and the MAT Project produced data on student attainment of the goals of the curriculum. The reports of the Mathematics Assessment indicate that students who have completed Grade 3 perform satisfactorily on items dealing with most of the objectives of the curriculum. Particular weaknesses were noted, however, on items dealing with students' knowledge and comprehension of the SI units of measurement and with rounding numbers, two areas which many teachers apparently do not cover in their teaching. Areas in which student performance was strongest included mastery of the basic facts of addition and subtraction, computational skills and the ability to interpret bar graphs.

Students who had completed Grade 7 did relatively poorly on items dealing with rational number concepts, geometry and measurement. The strongest performances at this level were recorded on items dealing with computational skills and knowledge of terminology and notation. As in the case of Grade 3, it was not surprising to find that students did poorly on those items on topics which many teachers indicated they had not considered in their classes. By the same token, it is undoubtedly true that those areas in which the achievement levels were highest were among those given the greatest degree of emphasis by teachers.

The truth of these remarks is perhaps best illustrated by the results obtained in the pilot-testing of the 300 items in the Grade 10 item pool for the MAT Project. In the case of those objectives which had not been taught by the majority of teachers, many of the results were quite poor. The mean percentage correct for the seventeen items in the pool which dealt with compound interest was 33.6 per cent; for the five items dealing with vectors, it was 35.2 per cent. Only one objective had a lower mean percentage correct, 29.4 per cent; this concerned the topic of geometrical constructions. The mean percentage correct for the entire pool of 300 items was 51.1 per cent.

Nine of the seventeen compound interest items had a mean percentage correct of less than 25 per cent, the 'chance' level. An example of such an item is shown below, with the correct response underlined.

'What will be the compound interest on \$1000 at the rate of 7 per cent compounded semi-annually for 7 years?
(Use the table at the end of the booklet.)'

a. \$1618.70
b. \$ 618.70
c. \$1605.80
d. \$ 605.80

Only 14 per cent of the students selected the correct response. Almost 40 per cent chose response c; approximately 25 per cent omitted the item entirely.

On the other hand, the results also reflect those parts of the Grade 10 curriculum which were being emphasized by teachers. Of the thirty-eight items in the pool with 75 per cent or more correct, thirty-seven were from the algebra section of the course. Although the curriculum guide calls for three content strands at this level (algebra, geometry and Consumer Mathematics), these results seem to indicate that teachers are emphasizing the algebra strand at the expense of the other two, especially consumer Mathematics.

Conclusion

A comparative analysis and discussion of the goals of mathematics education internationally is most appropriate at this time, when pressures for change in those goals and in the curriculum which emanates from them are being exerted. There has been a fairly high degree of disillusionment with, and movement away from, many of the excesses associated with the so-called 'new mathematics', which characterized the mathematics curriculum in many parts of the world for more than a decade. In its place, there is now underway, in many areas, a comparable movement toward increased emphasis on basic skills, minimal competencies and ensuring that students are exposed to the mathematics they will need in order to enter the world of work. Mathematics curricula must, of necessity, reflect these changing views of the nature, role and importance of mathematics in school programmes.

The 'intended' goals of the mathematics curriculum in British Columbia, as stated in the official curriculum guide, do reflect the influence of such views. They are in many respects similar to the goals and objectives of the mathematics curricula in the other provinces of Canada, and, for that matter, to analogous statements of goals from other countries such as the United States of America. One of the major assumptions which underpins the 'intended' goals of the B.C. mathematics curriculum is that all students, *up to and including those* in Grade 10, should follow the same basic programme in mathematics, and that one of the major components of that programme should be the study of the applications of mathematics in the day-to-day world of the consumer.

It is evident that there are discrepancies between the 'intended' goals of the mathematics curriculum of British Columbia and the 'implemented' or the 'realized' goals. Data obtained from both teachers and students indicate that there has been a lack of adequate communication of those goals, and that, as a result, the new programme has not been implemented as originally planned by the Curriculum Revision Committee.

It is not within the scope of this paper to speculate about the reasons for this lack of congruence between the 'intended' goals, the 'implemented' and the 'realized' ones. Neither is it the purpose of this paper to single out the case of British Columbia for special investigation. On the contrary, it is hoped that the example of British Columbia can serve as a case in point for others to consider in examining their own situations. The conclusion to be drawn from the B.C. experience is that the goals of the mathematics curriculum, as they are put into effect in classrooms by teachers and attained by students, may differ in significant ways from the goals as codified in curriculum guides. Moreover, any examination of curricular goals which fails to address the possible existence of such discrepancies between the goals as intended and the goals as implemented or realized will be incomplete because of that omission.

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Robert W. Morris

Report of a Meeting on the Goals of Mathematics Education[1]

During the week ending 23 May, 1980, there assembled in Paris, at Unesco's invitation, a group of twelve representatives of Member States, together with three observers, representative of the International Commission on Mathematical Instruction, the World Confederation of Organizations of the Teaching Profession and of the International Baccalauréat. Their purpose was to look afresh at the goals for mathematics education in the aftermath of the curricular upheavals of the sixties, the growth of informatics, computer science, calculators, data processing, etc., and the spread of mathematical usage into what formerly were called the 'descriptive disciplines', particularly the social sciences.

By way of inspiration and support, the participants had available to them the specially prepared papers which constitute the other chapters of this volume. They were provided also with a paper, entitled 'Some questions for discussion'. This paper had been prompted by the agenda for the Meeting.

The work of the Meeting was conducted partly in plenary session and partly in working groups, of which there were three in number. The findings of the Meeting follow. The wording used for all subsequent headings, below each of which the participants' conclusions are set out, is, in fact, the wording used to frame the agenda of the Meeting.

Mathematical Goals within the Context of General Education

The Use of Mathematics in Daily Life

Mathematics, today, permeates the whole of society, and its role would appear to be one of ever increasing importance as its help is sought in handling situations and problems which arise outside the field of mathematics itself. Mathematical methods are no longer the prerogative of scientists, engineers and technologists. They are increasingly being used to analyse individual behaviour and to study attitudes and trends in opinion within society as a whole. These developments serve only to increase the demand for mathematical skills – skills in quantification, in mathematical modelling, in simulation and in algorithmic techniques. Hence mathematics may be seen as an integral part of man's cultural, social, economic and technological environment, not only in its current form, but also in those other forms which will certainly develop as a consequence of the widespread capability of fast calculation.

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[1] Summary of a report of the Unesco Meeting of Experts on Goals for Mathematics Education; Mathematics Teaching and the Needs of Society (Paris, 19-23 May 1980).

Such considerations have important consequences for the goals of mathematical education. To impart the skills necessary to perform for example, the 'four rules' in decimals is no longer adequate – if, indeed, it ever was. The mathematics encountered in daily life does not appear in the form of the straightforward command of the textbook. 'Add 27.62 and 13.81'. Mathematics appears, rather, in the context of spoken language (or written), in physical situations, and in many transactions between individuals which characterize social life.

Mathematical information, too, is conveyed in a variety of ways. Economic information, for example, which bears heavily upon the circumstances of living and which individuals need in order to make decisions is often displayed in formats peculiar to mathematics itself; in tabular form, in graphical form, or in the form of an algebraic statement. This fact is, of itself, as things are sufficient to debar many individuals from access to economic information.

Mathematics, it is said, provides training in 'problem solving'. Indeed one of the more prominent goals of mathematics education has always been to impart skill in that highly prized 'ability to solve problems'. In the real world, however, there are, broadly speaking, two problems. One is to identify 'the problem', surrounded, as it usually is, by masses of irrelevant information. The other is to find the appropriate tecnhique for solving 'the problem', when it has been identified. School mathematics, unfortunately, gives little training in solving these two broad problems. In the first place, 'the problem', in the mathematics class, is immediately apparent. Stripped of all 'noise', it stands out in the open, like a windmill to tilt at. Indeed, to present it otherwise would be regarded as 'unfair'. Secondly, it is conventional to offer the class 'problems' when they have mastered a particular technique, as, for example, the technique of 'solving simultaneous equations in two unknowns'. The techniques required to solve school problems can therefore be inferred from the routine work which preceded the exercise on 'problems'.

It is no wonder, therefore, that many people in the course of their daily lives, find it difficult to:

- sense a situation where mathematical methods are applicable;
- associate 'a problem' of some kind with a mathematical 'model' of the problem; or
- recognize the gain in efficiency of thought and effort which comes from thinking about the problem in mathematical terms.

Evidence is accumulating which suggests that classrooms, as we know them, do not offer the most suitable milieu for training in problem solving. Children need to be placed in the denser thickets of situations where their mathematical equipment has to be used to cut a way into the open. This type of teaching takes time. However, experience has shown that after a year or so of learning in this way, children do begin to think mathematically and do begin to appreciate the advantages of possessing some simple mathematical equipment. There is some evidence also that this type of teaching helps to promote enjoyment of mathematics and to remove the fear which has, in the past, led many people to turn away, physically and psychologically, from applying mathematics to situations which crop up in daily life.

The Applications of Mathematics

Applications of mathematics pervade the environment, and every individual will encounter the use of mathematics in three broad contexts:

- in the context of his private life;
- in the context of his working life; and
- in the wider context of the social, economic and political life of the country of which he is a citizen.

Some basic mathematical skills are needed simply to 'get by' in life. Others are needed and used at one's place of work. The scope of these, obviously, depends upon the job. Some jobs call only for modest mathematical skills. Others can be immensely demanding. In the wider context of a person's social, economic and political life, an intelligent appreciation of 'what is going on' is scarcely possible unless a reasonable level of mathematical literacy has been attained. Here, mathematical insights help the individual to become an open-minded, yet critical observer and participant in the wider life of the community and of the nation.

What are the implications for mathematical education of these three broad contexts of use?

The Meeting agreed that there is no fundamental difference between the nature of the mathematical education required for daily life, for its use at the place of work, or for observing and taking an intelligent interest in the social, economic and political life of the community and of the nation. In all three contexts, the individual has to tackle non-mathematical situations of varying complexity. In all such situations, the mathematics involved is neither uniquely determined nor visible from the outset. No ready-made amalgam of mathematical insights, concepts or skills will suffice to cover the whole variety of situations which can occur in the three contexts described above, or in the continuing changes of these over time. On the other hand, no situation can be properly appreciated, weighed up, judged or unravelled without the skills and insights needed to cope with the mathematical components of the situation.

Such considerations led the Meeting to conclude that no mathematical topic is important in itself. The potential importance of a mathematical topic derives from its potential capacity to serve the needs at issue, whatever they may be. This calls in question the conventional method of defining curricula only in terms of topics, concepts and skills. The Meeting believed that mathematical curricula should also be defined in terms of situations to be appraised, judged, analysed, and determined, in conformity with the realities of the world we live in. It recognized, however, that to propose so radical a change would be unrealistic. It accordingly recommended a gradual approach to defining curricula in terms of different situations that might lead to considering relevant mathematics. The situations should be chosen and presented in such a variety as would ensure many different aspects of mathematics being called into play. Not only should situations be considered where mathematics is required by students for use as a tool, but also situations where the mathematical ingredients can be observed from without, as an activity with social, historical philosophical and ideological relationships and characteristics.

In particular, the Meeting made three specific recommendations for mathematics education as follows:

1. A major aim of mathematics education in *primary schools* should be to develop the ability to use numbers in situations that arise in daily life. As the students progress in their studies, the scope and complexity of the applications should increase.
2. Although it is believed that, in the long term, *secondary school* curricula ought to be reorganized to include situations from everyday life and from other fields of study, it is necessary to think in terms of progressive steps which are feasible, having regard to contemporary school structures and to the existing

human and material resources that are available. Hence, as a first step, part of secondary school time should be allotted to project work involving an integration of the sciences and mathematics.

Such a change in the goals of mathematics education will ultimately call for changes in existing methods of evaluating the performance of students. This need not prove an obstacle to curricular developments taking place in the directions recommended. There is a body of experience of assessing essays and compositions on the basis of the approach adopted, the reasoning, the clarity of presentation and the elegance of the wording. In the same way, examinations could be set where an open, complex, non-mathematical problem could be solved at different levels of sophistication using varying levels of mathematics. The Meeting therefore recommended that examinations include open-ended problems which should be evaluated according to the approach and adequacy of the solution. It was further agreed that research was called for into the design and the assessment of open-ended problems.

The Contribution of Mathematics to Developing the Personality

It is possible to teach mathematics in ways that are conducive to the personal development of attitudes, skills and powers of appreciation.

The particular personal attitudes which can be engendered are:

- those of self respect;
- those of self-reliance, including a willingness to take responsibilities;
- those which are conducive to self-discipline;
- those of consideration for and ability to co-operate with others;
- those of patience, particularly in the pursuit of the solution of a problem.

The particular skills which can be inculcated are those:

- of observation;
- of interpretation;
- of communication.

And the powers of appreciation which may be nurtured are those:

- of the beauty and elegance of some mathematical proofs;
- of the relationship of mathematics to life.

Mathematics can also help to bring out the natural creativity of the child, providing him with the opportunity to use this creativity in the discovery of abstract concepts that are new to him. It can also make an important contribution to the development of the personality of the learner by offering him a language that precisely expresses even the most abstract ideas. In its use of graphical representation, it can impart the power to convey highly complex information in a clear and concise way.

Among the cognitive abilities that can be developed, mention must be made of:

- spatial perceptions, and the appreciation of the properties of configurations in two or three dimensions;
- numerical understanding;
- ideas of measurement and of its approximate nature, giving rise to the concept of continuity;
- the representation of numbers in graphical form and by drawing to scale, as, for example, in maps, charts, plans, etc.

The proper learning of mathematics will arouse and develop the child's powers of creativity. It will often lead him into new ways of solving problems.

Problem-solving is a complex mental activity. It would appear to involve, first of all, the identification of the relevant 'bits' of information. Actions then have to be taken in accordance with the previously made decisions. Finally, the mathematical outcome has to be reinterpreted in a manner which is consistent with the situation which is being considered. As Polya has pointed out, when facing a problem, the learner learns to ask pertinent questions. Other important factors in problem-solving are the exploration of all the possibilities and inspired guessing. Both should be actively encouraged in teaching.

Closely aligned to problem-solving, and often one of its main ingredients, is mathematical 'modelling'. This is a process in which the learner makes use of mathematical symbols to represent the measurable quantities that exist in reality. Real relationships can then be expressed as mathematical relationships. These mathematical relationships are then susceptible to mathematical manipulation, in the course of which a 'solution' may or may not be found. Once a problem is 'solved' in the realm of symbols, the learner returns again to reality to interpret the results.

Some of the elementary laws of physics, such as

$$pv = k, \quad s = kt^2, \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

etc. provide simple examples of mathematical 'models'.

The contributions of mathematics to intellectual development include a fostering of the growth of abilities such as the ability to explore, to search for information, to make decisions, to abstract, to generalize, to analyse, to synthesize and to classify. As for the contribution of mathematics to human achievement, this is evident everywhere, in all civilizations and in all cultures.

Pedagogical factors to be considered in determining goals

There would appear to be strong evidence to support the view that older children are often at odds with their teachers about what they are expected to learn. There are many instances in which a secondary school pupil, knowing the type of work he is going to do after leaving school, fails to see the relevance of the mathematics syllabus to his future needs. In such cases, an effort should be made by the teacher to recognize this and to find relevant appreciations of mathematics to the pupils chosen career. On the other hand, the validity of obtaining qualifications must be recognized as a necessary reason for the treatment of certain topics, since the real world is a place where examinations are held and the pupil should be prepared to undergo them. Syllabuses cannot, of course, cater for every different type of job which the students will go into. It would, however, be helpful when syllabuses are being drawn up to incorporate examples of the mathematics used in a variety of the commoner vocations.

A deeper knowledge, on the part of the curriculum planners, of the community life surrounding the school would also have a positive effect on the contribution of school mathematics to the enrichment of personal life. In particular, curriculum planners and the teachers should take account of any community action programmes. They should look into these with a mathematical eye in order to identify the specific problems and to cater for the mathematical skills needed.

The short-term needs of the learner point to the importance of good teaching in order to make mathematics seem to the pupil a pleasurable and worthwhile task. Such teaching challenges the natural creativity in the student, and the judicious choice of tasks will permit him to explore different ways of overcoming difficulties.

Pleasure derives also from using mathematical knowledge in many different situations in and out of school: sometimes when reading newspapers, in applications to physics and to other subjects, etc. Most children come to realize that mathematics is useful, even if the realization is induced by parents, teachers and society in general.

Linguistic Considerations in Determining Goals

Two different situations arise in considering language in relation to the teaching of mathematics: first, when the teaching is done in the mother tongue; second, when the teaching is given a second (or third) language. The traditional way of starting is to teach how to count, and how to write numbers, associating numbers often with dots on a piece of paper, or, less often, with actual objects. Later the skills involved in adding, subtracting, multiplying and dividing numbers are taught. Such teaching is almost invariably dissociated completely from the natural world of the child, where, in practice, the numbers are embedded in the words used by the community to which the child belongs. The consequence of this is that the child does not associate numbers with life around him. And teaching is done without regard for the natural way in which children use numbers in their own lives. This traditional way of teaching is encouraged by the textbooks which treat numbers in a manner which is unnatural to the child and in a way that does not fit the children's experiences of numbers in real life. Teachers tend to keep closely to textbooks. So, they do not usually talk about numbers or measures in a way which is familiar to the child in his out-of-school experience.

When teaching is done in a second language, the child not only faces the same problems that have been described above; he also has to learn the symbols and attach to them words he has never met before in his home or village life. In some cases, nonsense can arise. For instance, in Kiswahili, the hours of the day are counted from six o'clock in the morning, so that 'twelve' is hour 'six' and 'two' is hour 'eight'.

Third, in relation to language, it is necessary to remember the remarkable economy of usage in the mathematical register. The symbols of mathematics serve for a wide variety of linguistic usage. Take, for instance, the symbol '+'. This symbol is used where many different physical situations ensue. It is used, for example, when two (or more) collections of objects are 'put together' to make one collection. It is used when two or more collections are not 'put together' but 'the total' is required: 'How many goals were scored altogether?'. It is used for 'and then' – when, for example, I walk 250 meters 'and then' another 30 meters. It is used in relation to growth, in relation to extension, and many other physical situations. It is therefore very important for children to be given a rich variety of linguistic situations in order to learn the appropriate mathematical operation that different situations require. Consequently, good teaching will at first make use of the child's natural language, and will only gradually replace the words with the symbol (instead of the other way around). In this respect, chemistry teachers are perhaps more perceptive than mathematics teachers. They will spell out the name of substances like carbon dioxide for a long time before they write the formula 'CO_2'. Teachers who have to teach in a second language have exceptional difficulty in presenting mathematics in a natural context, relating numbers, for example, to the measurement of time and weight and to money and so forth. They have to be acutely aware of the linguistic difficulties of their pupils. They need to be extremely careful to avoid using words and structures of sentences which are unfamiliar to the children. Such teachers need linguistic guidance as well as help in the teaching of the subject. In second language situations, it is important both to build mathematical language into the language lesson and, conversely, to teach language in the mathematics lesson. Textbooks influence the teacher strongly.

Developing countries have particular textbook problems. Sometimes they adopt a book written for another country, with particularly unfortunate results. Sometimes they adapt a book of another country. This removes some but not all of the difficulties. What they need most of all is to be encouraged to develop their own capacity for producing textbooks. The group noted that some countries develop their textbooks for primary education, using a team approach and involving experts in language and in teaching as well as in the subject itself. A recent conference held in Accra recommended that mathematics should be taught to young children in the vernacular. The implementation of such a recommendation would imply an enormous effort in local textbook development.

The Mathematics Syllabus as a Reflection of How Children Learn

The Meeting found Margaret Brown's paper (pages 23-40) very helpful in contemplating this topic. It would seem necessary to create among teachers a better awareness of the presence of different levels of understanding of all mathematical concepts. Participants believed that it would assist teachers if they had available to them tests which would help them to measure the various levels of learning reached by individual children. This suggests a need for further research of the kind described in Margaret Brown's paper. Mathematics traditionally has been regarded as a sequential subject. The findings of research would seem to qualify this long-held belief. Some concepts are sequential, others are not. As with life, variety is the spice of mathematical experience. This suggests a spiral approach to syllabus building.

Definition and Evaluation of Goals and Translation of Goals into School Curriculum

Strategies for Determining National Goals

The curriculum should reflect what the people of a nation think, feel, believe and do. Therefore curricular development should be an expression of a wide range of social, political and pedagogical goals. Curriculum development may best be conceived as attempts by professional educators and other concerned groups in a nation to promote cultural synthesis. In a fundamental sense, curricula development at a regional or a community level should represent a comparable mixture of social, political and pedagogical goals, realized through a parallel process of curricular development. So, in what follows, the processes of goal determination and of curricular development will be discussed in terms of an educational authority, irrespective of whether the endeavour is a national, regional or a local one, since the same processes apply.

For purposes of discussion, it is possible to identify two types of decisions which have to be made by schools:

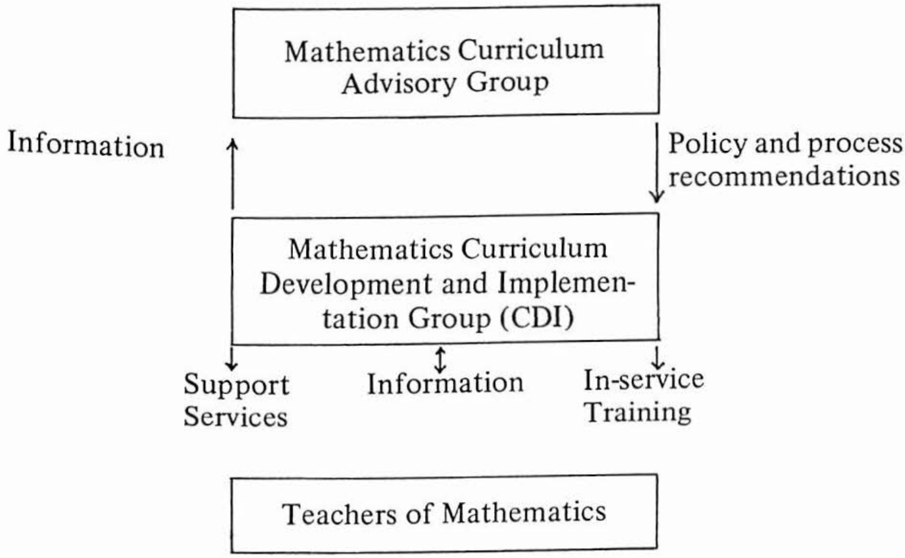
- the first type are those decisions which concern the structure of the school and its functioning, other than those factors which are specific to mathematics (they will reflect the administrative structure of the school, the length of the school day, the language requirements, etc.);
- the second type are those decisions which concern the structure of the school and its functioning but which are endemic to mathematics teaching and learning (they will reflect teaching methods, instructional materials for mathematics, the training of the teachers in mathematics, evaluation materials for mathematics, etc.).

This division helps to identify the relationship between the decisions which affect mathematics and those which affect the rest of the school's work. Decisions of the second type, which concern mathematics teaching and learning, should seldom be made without regard to the first type, since most decisions are an interactive combination of effects.

In the context of decision making, it is to be noted that most curricular changes in mathematics have an impact on, and require the co-operation of, all groups represented in the school community. This co-operation thrives best when there is a quick, easy and open flow of information about the impact of change on students' learning, on teachers' teaching and, indeed, on the entire community, within and without the school.

It should also be noted that if curricular development is left solely to the judgement of the professionals in mathematics education, the decisions may ignore other segments in the community directly and dramatically affected. Curricular development processes must deliberately employ means and methods that involve information about the child, about schooling and about the role of mathematics. This information must be derived from sources beyond the professionals in mathematics education, since the decisions which are made have the potential of powerfully influencing what lies beyond mathematics in the schools.

That said, the processes involved in the determination of goals, in curriculum development and implementation, and in the evaluation of the curriculum are best done if a central group is made responsible. For purposes of discussion, this group may be called the Curriculum Development and Implementation group (CDI). An advisory group to the CDI should provide the judgements and wisdom on policy and processes which the CDI may require. The relationships between the CDI, the advisory group and the teachers of mathematics in the system may be illustrated as follows:


Mathematics Curriculum Advisory Group
Information
Policy and process recommendations
Mathematics Curriculum Development and Implementation Group (CDI)
Support Services
Information
In-service Training
Teachers of Mathematics

Goal Determination

The various groups that have an interest in the perfomance of the schools should be involved in the identification and formulation of the statement of curricular goals of the school mathematics programme.

Since the educational task is one of building for the future of the entire society, every sub-group of the society has a responsibility to participate in the identification of goals. Some groups that must be involved in the determination of goals are the teachers, the parents, the students, mathematicians, the employers, the employee organizations, educators and the political authorities.

Involving the various groups in the process of determining goals may yield conflict. Some groups may be very conservative, wanting the schools to keep on doing just what they have always done in the past. Others may be committed to the schools' serving purposes unlike those respected in the past. CDI must realize that some conflict is to be expected and it must be willing to guide the resolution of such conflict. An important by-product of the process of reaching a consensus is that the process serves to educate members of widely different groups in the community about the nature of the school and the problems associated with education.

Goals cannot be identified in vacuo. The process is one of making an informed judgement based upon factual information about the current status collected from a variety of sources. Among these sources are:

- School reports (i.e. performance data concerning pupils, enrolment data, cost figures, curricular documents, etc.);
- Statements of national purposes and goals;
- Economic and demographic characteristics of the population served by the schools;
- Employment data and forecasts.

CDI should provide information on the current status to the advisory group in a form which can be readily interpreted. CDI must recognize that information can be expensive to collect. It should, where possible, use other sources of information, rather than collect it directly. Furthermore, CDI must exercise judgement on how much information the advisory group will need, if the information is expensive to collect. In the long run, curriculum development should recognize its own transitory nature. Its evaluation processes should be designed to provide a continuing source of current information that can be used for the purposes of goal determination in subsequent years.

The process of identifying goals should take into account opinions on 'what-ought-to-be' collected from a variety of sources. The sources should include teachers, parents, mathematicians, employers, employees, political authorities, educators and students. Although survey methods are appropriate for collecting such opinions, CDI needs to recognize the practical limitations and the cost of these methods, and, if resources are limited, should then use more information methods of collecting opinion, such as interviewing representative samples or asking the groups concerned to collect the information required.

Goals should be formulated in terms of the needs of the individual and the needs of society. Questions that should be considered in identifying goals are:

- Do the products of the mathematics programme have the basic literacy in mathematics to deal with the everyday use of mathematics?
- Does the mathematics programme provide learning experiences of sufficient breadth and depth to allow those students who so desire to continue their studies in science, mathematics and other fields?
- Do the products of the mathematics programme have the understanding and the skills requisite to functioning in and contributing to important vocational and economic sectors of the society?

CDI should formulate questions bearing on the unique circumstances of member schools for the Advisory Group to consider when deliberating about goals. The closer the questions are to the characteristics of society community, the more useful will be the wisdom of the Advisory Group. Statements on goals should specify the curricular actions that can be done. The process of identifying goals is of little value, or use, if the resulting statements are as general as: 'The teachers shall do a better job of teaching mathematics'. This statement is faulty because it contains no indication of the type of actions that need to be taken by CDI. Although it is universally true, and no one would disagree with the intent, it is not specifically related to any current status information, nor to any opinion on 'what-ought-to-be', nor to any discrepancy between these two.

In order to sharpen and refine statements on goals to a usable, do-able form, it is suggested that the responsibility for preparing statements on goals should not be exclusively that of the Advisory Group, but should, at the least by shared by CDI.

Translating Goals into Objectives and then into Curriculum

CDI is responsible for transforming statements on goals into specific objectives, for instructional materials (including texts) and for teaching methods. The objectives, the instructional materials and the teaching methods are inter-related, and they should be mutually compatible. CDI may elect to have the goodness of fit between the statements on goals and the resulting objectives, instructional materials and teaching methods verified by independent experts in mathematics and in pedagogy.

The process of transformation should progress through three distinct phases – planning, piloting and diffusing.

The *planning phase* is crucial to the success of the process of transformation. Curricular goals are typically of a broader, more general character than are statements of objectives. Objectives are small, more specific requirements, around which planning is organized. Objectives should be examined to see whether they require (and they may not):

- New instructional equipment;
- New textual materials;
- Different teaching methods;
- Changes in evaluation procedures;
- Modifications of other factors in the school programme or changes in other discipline areas.

If the objectives require one or more of these changes or modifications, specific planning should identify what is to be done, by whom it is to be done, the resources needed, and by when it should be done. If the planning is specific to the objectives, a sense is felt of progress being made toward achieving the goals and to evaluating the process.

Careful planning should be sufficiently comprehensive to provide advance warning of other actions that need to be taken. For example, a set of new curricular materials may require a modification in teaching methods. If so, an in-service programme may be required. If this is the case, planning should include identifying the in-service leader, arranging the time, place and procedures for in-service education and an evaluation of the in-service activities. Another element of careful planning in such a case would be the modification of pre-service professional training so that new teachers need not immediately be required to participate in in-service training in the use of the new materials.

The *pilot phase* is critical since it decreases the risk of subsequent modification. The lessons learned during the pilot phase of curriculum change can make for success in implementation. If in the pilot phase a change is found to be ineffective in meeting a curricular goal, modification is much less expensive than if all the teachers and students are involved. The evaluation of the pilot stage must be a measure of the success of the programme in meeting the curricular goals. Evaluation of the pilot stage will indicate whether or not the programme change should be implemented by all teachers in all schools. It will also indicate whether modification and another pilot experiment is needed.

The pilot testing of new curricular materials or methods often creates an effect on teachers and pupils of excitement and interest, simply because the experience is new, different or distinctive. These attitudes can contribute to the students' and teachers' working harder and doing better than might be expected in the more normal, regular school setting. The inflated performance, often called 'the Hawthorne effect' in the literature of evaluation, must be allowed for in evaluating the pilot stage, since it may need to be compensated for in planning the diffusion.

For each of the phases – planning, piloting, and diffusing – it is necessary to take into account the impact upon the pupils, the teachers, the school and classroom environment, and, indeed the impact and implications for the entire school programme including the other discipines taught in school.

CDI should at all times remember that it has a major and primary role in making certain that curricular change works. The magnitude of the task means that CDI must be willing to find and use expertise from a wide variety of sources at many points in meeting this responsibility. And they must be willing to delegate responsibility to other professionals including teachers.

Evaluation

Evaluation is the collection and use of information. Whatever is done will be of appreciably better quality if appropriate information is used to guide the decisions involved. Thus, evaluation is not reserved for the final stages of the process of change nor is it limited to the evaluation of pupil performance. The fitting of the goals to national needs must be evaluated. The planning stage should be examined to determine whether it is sufficiently comprehensive and realistic. The piloting must be considered in terms of its potential for revealing such key elements as will need attention for subsequent diffusion and for the success of the innovation in meeting goals. The diffusion process needs to be evaluated in terms of its efficiency and cost. These, and other factors, must be accounted for if the evaluation is to be thorough and useful. The evaluation procedures are most easily accomplished if the objectives are well-stated.

The process of identifying curricular goals, previously described, may yield goals of many different types and varieties. Some may involve curricular modifications that can be evaluated simply by giving a test at the end of a new unit. Others may be more complex and may take a much longer period of time to evaluate. For example, if the goal is 'to produce individuals able to function more effectively in using mathematics in farming', then the process of evaluation cannot be completed until the products of the school are out on the job. In short, written achievement tests do not necessarily provide all the evaluative information. In particular, it is unwise to rely upon those achievement tests which are suitable for decisions on university entrance to provide information on those facets of the curriculum which were designed for other purposes. The evaluation procedures must fit the goals of the programme. Some important goals are not observable or measurable in the short term.

Identification of Problems Requiring Further Action

In the light of its findings, the Meeting concluded its work with an identification of those general problems which appeared to warrant further action by Unesco or by Member States or both. It was generally agreed that the most urgent problem was that of teacher education, both pre-service and in-service. In this context, the needs of primary teachers were paramount, though, clearly the effectiveness of secondary teachers is crucial to the supply of students to the primary teacher training colleges.

Bearing in mind the need to make mathematics in school more reflective of its role in the real world, the Meeting believed it was necessary to make studies of the part played by mathematics in the life of the community and in society generally. Such studies would generate examples of mathematics in action which could be used to enrich curriculum.

The need for teachers generally to appreciate the fact that there are different levels of understanding of mathematical concepts points to the desirability of undertaking further research of a similar kind (described by Brown) in different countries of the world.

The role of language in the learning of mathematics is crucial. Unless children understand the words used to convey the mathematics, they will not understand the mathematics itself. The problem is a real one in all countries. It is particularly acute in countries where the medium of learning is not the mother tongue. The Meeting was strongly of the opinion that much more attention should be paid than hitherto to the interactions between language and learning in mathematics.

The Meeting was also deeply concerned with the plight of the world's millions who have never had the opportunity of a formal schooling. In a technological era, these people cannot escape the need for some mathematical perceptions within the context of their daily lives. It was accordingly believed to be essential that attention should be given to the problems of imparting numeracy as well as literacy in programmes of non-formal education.

Biographical Notes

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MARGARET BROWN is Lecturer in Mathematics Education at Chelsea College, University of London, United Kingdom. After studying mathematics as an undergraduate at Cambridge she took a postgraduate certificate in education at the London University Institute of Education, and taught in secondary schools before joining the staff of the Centre for Science and Mathematics Education at Chelsea. She has worked for some years in the area of mathematical concept development in secondary school children on which she has published several papers. She is also interested in the teaching of probability and statistics and in the learning problems of low attainers.

SVEN HILDING has been His Majesty's Inspector of Mathematics in Sweden since 1966. He started his mathematical career as a scholar of the late Professor J. von Neumann at the Institute for Advanced Study, Princton, New Jersey, in 1947. He was later assistant professor in Mathematics at the University of Stockholm and head of the Math Department at Teacher's College in Stockholm. His main interests concern curriculum development and research in teaching methods in Mathematics.

ED JACOBSEN is Programme Specialist in mathematics education in Unesco's Division of Science, Technical and Vocational Education in Paris. After completing an M.Sc. in mathematics at the University of Wisconsin, Madison in 1958, he lectured for six years at the then Robert College (Istanbul American University). Returning to the United States, he obtained a Ph.D. at the University of Kansas. After lecturing for a few years in the United States and Spain, he joined Unesco, serving in faculties of education at the University of Nairobi and the University of Botswana, Lesotho and Swaziland before being transferred to Unesco Headquarters in Paris where he is responsible for mathematics education.

MURAD E. JURDAK was born in 1943 in Lebanon where he obtained his B.S. (1966) and M.S. (1968) from the American University of Beirut (AUB). He obtained his Ph.D. in mathematics education at the University of Wisconsin, Madison. Dr. Jurdak is currently an Associate Professor of Mathematics Education and Chairman of the Department of Education of AUB. As a faculty member of the Science and Mathematics Education Center (SMEC) of AUB since 1973 and its Director 1976-78, Professor Jurdak was involved in mathematics curriculum projects in the Sudan and Saudi Arabia and also in teacher training in several Arab States. His research interests include development of logical thinking, readability of mathematics textbooks and curriculum research.

ROBERT MORRIS has had a varied career in education. After eleven years of experience as a schoolmaster, he joined the ranks of Her Majesty's Inspectors of Schools, from which he served, on secondment, as Educational Attaché at the British Embassy in Washington D.C., as joint head of the Curriculum Study Group in the Department of Education and Science, as joint Secretary to the Schools Council for Curriculum and Examinations, as Director of the Centre for Curriculum Renewal and Educational Development Overseas (CREDO), as Director of the Curriculum Division of the Centre for Educational Development Overseas (CEDO), and, finally as Director of the Continuing Mathematics Project in the University of Sussex. He was appointed Editor of *Studies in Mathematics Education* in 1979.

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RUTH REES is Senior Research Fellow in Mathematics Education at Brunel University, United Kingdom. She graduated in Physics at the University of Wales in 1950 and then held various teaching posts as head of Departments of Science and Mathematics in schools. During the period 1962-1967 she furthered her interests in mathematics by teaching at the University of Calgary and completed an M.Sc. in the field of Rheology. Her teaching experience has covered the primary, secondary and tertiary sectors. Since 1970 she has researched into the problems of learning and teaching mathematics and now leads a small team carrying out investigations from the primary level through secondary to continuing education. During this time she obtained a Ph.D. at Brunel for an exploratory study of mathematical abilities. She has wide interests and commitments in the field of mathematics education including the numeracy needs of adults and the needs of the mathematically gifted students.

ANDRÉ REVUZ, a former student of the Ecole Normale Superieur and Doctor of Science, is a Professor at the University of Paris VII. For the last twenty years he has devoted considerable effort to the problems involved in teaching mathematics. From 1968 to 1979 he directed the Institut de Recherche sur l'enseignement des Mathématiques (IREM) (Paris-Sud). At present he is responsible for graduate studies in Didactique des Mathématiques at the University of Paris VII.

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FREDERIK VAN DER BLIJ is Professor of pure Mathematics at the University of Utrecht, Netherlands. He was born in Leiden, Netherlands in 1923, and took his Doctors degree at the University of Leiden in 1947. He taught in secondary education for several years. His teaching load embraces number theory and analysis for students in pure mathematics as well as courses in applied mathematics for non-mathematicians and special courses connected with teacher training. Until its demise in late 1980, he was Director of I.O.W.O., an institute for the development of mathematical education in the Netherlands.

HERBERT VOCKENBERG is a scientific staff member of the Mathematics Division in the Institute for Education in Mathematics, Natural Sciences and Polytechniques of the Academy of Pedagogical Sciences of the German Democratic Republic. Originally trained as a mathematics teacher, he taught at secondary school and teachers' training institute level and worked as subject adviser and reader for study materials of mathematics in the field of teachers' distance learning. He then went on to obtain his degree of *Doktor* from the Academy of Pedagogical Sciences in 1972 in the field of heuristics. His particular interests include mathematics education at the *Abitur* level, non-compulsory mathematics for interested pupils, and the application of mathematics to social practice and other sciences.

AVRUM ISRAEL WEINZWEIG is Associate Professor of Mathematics and director of the Master of Science in Teaching programme at the University of Illinois, United States. Born in Toronto, Canada, he has a B.A.Sc. in Engineering Physics from the University of Toronto. After working as a Research Physicist at the Weizmann Institute of Science, Israel, and as Assistant Chief Geophysicist at Weiss Geophysical, Canada, he went to Harvard University where he obtained a M.A. and a Ph.D. in Mathematics. He has worked in Algebraic topology, category theory and applied mathematics. His main research interest now is in mathematics education, particularly the development of mathematics concepts in children. Twice elected President of the Illinois Council on Mathematics Education, he is at present a member of the International Mathematics Committee for the Second International Study of Mathematics, a member of the United Nations National Committee and Vice President of the Commission internationale pour l'étude et l'amélioration de l'enseignement des mathématiques.

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